Funding under Borrowing Limits in International Portfolios[1]

Tommaso Trani[2]

The Graduate Institute | Geneva

Job Market Paper

This version: February 14, 2012 (polished more recently)

Abstract

I develop an open economy portfolio model to study how leveraged investors' wholesale funding affects the international transmission of shocks. Under binding borrowing limits, there is a link between the international investment positions of integrated economies as investors diversify the asset side of their balance sheets. Building on this mechanism, I introduce the liability side, allowing investors to sell domestic and foreign bonds and capturing changes in counterparty risk in a stylized way (i.e., debt-to-asset ratios are specific to each borrower and time-varying). I model and parameterize these ratios, conditional on portfolio choice. I can solve for portfolios taking advantage of the link between assets and liabilities which is implied by the borrowing constraints. Equilibrium portfolios feature *home funding bias*, which is justified by a crucial interaction between the terms of trade and the tightness of the borrowing constraints. Dynamically, this interaction implies that the source of debt which is most sensitive to shocks is foreign funding. In fact, any shock creates a wedge between the cost of funding in different countries; the value of collateral must adjust accordingly through asset prices. Yet, asset prices are mainly affected by financiers' concerns about counterparty risk: impact effects are deep and in line with the terms of trade effect. Combined, these effects have somehow novel implications for the net foreign asset positions. The cumulative effects have instead more mixed results on fluctuations.

JEL classification: E21, F32, F34, F41, G15.

Keywords: borrowing limits, counterparty risk, financial flows, international financial markets, international lending, macroeconomic interdependence.

[1] The preceding draft of this paper was titled "Trade in secured debt, adjustment in haircuts and international portfolios". I am indebted to Cedric Tille for his unique, careful guidance, and I am grateful to Rahul Mukerjee for his advices and support. I also thank for comments Philippe Bacchetta, Isabella Blengini, Giorgio Calzolari, Salvatore Dell'Erba, Christian Friedrich, Mathias Hoffmann, Julia Schmidt, Sergio Sola, Natascha Wagner and seminar participants of the First Sinergia Workshop in Lausanne, the Graduate Institute of Geneva, the University of Zurich and the XII Doctoral Meeting in International Trade and International Finance at Bocconi University. Finally, I gratefully acknowledge the helpful hints received by Enrique Mendoza and Alan Sutherland. All remaining errors are exclusively mine.

[2] Contact: Department of Economics; 11A, Avenue de la Paix, CH-1202 Genève; tommaso.trani@graduateinstitute.ch.

1 Introduction

In this paper, I study how wholesale funding affects the international transmission of shocks. I seek to understand whether the liability side of investors' balance sheets has an independent role in the transmission of shocks across countries and how it contributes to the mechanism highlighted by recent literature. According to this literature, the international transmission mechanism of shocks works through the asset side of investors' balance sheets. Agents invest across borders in accordance with the borrowing constraints that they face. As a consequence, the constraints themselves translate into transmission channels.

Along these lines, I develop a two-country model where leveraged investors can potentially borrow from both local and foreign financiers, who are concerned about counterparty risk. Specifically, I build on the international transmission mechanism under leverage constraints proposed by Devereux and Yetman (2010) and obtain a framework which is new in two respects. First, the endogenous portfolio choice is not limited to assets that can be used as collateral; it also involves the international diversification of funding, under the constraint imposed by that collateral. Second, counterparty risk is evaluated in a stylized way through the debt-to-asset ratio, which determines how residents of a given country can borrow against a given amount of pledged collateral. Debt-to-asset ratios move with asset prices, conditional on portfolio choice.

These two characteristics of cross-border funding derive from as many historical developments. One is the tendency of a given financial institution to borrow from other financial intermediaries by means of standard secured loans. This tendency has been increasing since the 1980s, and - as the financial literature has often observed - under certain cirrcumstances it can create or worsen economic crises. The other development is the globalization in banking. For example, Goldberg and Cetorelli (2010) find that global banks have progressively modified the international transmission mechanism through both cross-country transactions and foreign offices and subsidiaries. My focus here is to capture how the globalization in banking and the practice of borrowing against collateral interact in international financial markets. Notable examples are the cross-border transactions involving repurchase agreements (repos) and asset-backed commercial papers (ABCPs).

Therefore, it is natural for me to refer to Devereux and Yetman (2010). Their important work highlights how the international positions of leveraged investors can generate financial accelerator effects in open economy[2]. Their two-country, one-good model features investors who purchase both domestic and foreign claims and who, simultaneously, pledge the resulting portfolio as collateral. The model is symmetric, so investors act in the same way in both countries. It follows that collateral portfolios in each country are exposed not only to domestic shocks, but also to foreign shocks. A given idiosyncratic shock affects both local and foreign portfolios, leverage constraint must tighten and the shock is transmitted

[2] See also Krugman (2008).

across borders with macroeconomic consequences.

This one-good framework implies that liabilities can be of one type only. As a consequence, local and foreign investors sell the same bond and the international integration between markets for debt securities does not have a clear role. Yet, the observed flows of interbank deposits between financially integrated countries seem to be substantial. In Figure 1. I aggregate the cross-border assets and liabilities of various advanced OECD countries. I consider BIS Locational Banking Statistics because I do not focus on multinational banks and the locational data are recorded using the *residency* principle[3], which is the same principle used for recording and organizing balance of payments statistics. Using World GDP as a scaling variable, I compare total external assets (assets vis-à-vis all sector) with external interbank deposits (liabilities vis-à-vis other banks).

The main message from Figure 1. is twofold: quantitatively, cross-border interbank deposits are almost as important as total external assets; the behaviour of the two series is quite similar. Indeed, the globalization in banking has shaped international flows especially during the last decade (Goldberg, 2009), although it has abated a bit after the 2007-2009 financial crisis. Part of the increase in global banking is certainly connected with the introduction of the Euro, and it is currently stronger across Eurozone countries than elsewhere (look at the second panel). But global banking is strong also for non-Eurozone countries (third panel).

In a concomitant and instructive lecture, Shin (2011) studies these type of cross-border banking flows, global funding in particular[4]. Among other things, he considers a breakdown of banking flows by the currency of denomination, emphasizing how the leading role of the dollar has shaped the recent flows between, say, a German bank and a U.S. bank. As shown by McGuire and Von Peter (2009), the role of the dollar motivates a general currency breakdown of international bank funding because it helps detect maturity and currency mismatches between asset and liability sides of balance sheets (Figure 1-A. in appendix A.3). Clearly, these mismatches start to bite during times of distress in international markets.

Coherently with this evidence, I introduce cross-border funding as follows. Leveraged investors sell home bonds to local lenders and foreign bond to foreign lenders, so that borrowing is from two different sources. Under binding borrowing limits, the size of this bond portfolio cannot be greater than the value of the total amount of pledgeable assets (the equity portfolio). Each international investor faces a *unique* constraint, which is the reason why the bond portfolio is endogenously chosen. In addition, the overall portfolio choice problem (involving claims and liabilities) can be simplified, taking advantage of the fact that the collateral constraint represents the dominant link between the two sides of investors' balance sheets.

[3] Note also that the BIS Locational Statistics include all interbank transactions made "on a trust basis" (BIS, 2008; p. 5) and in general collateralized transactions such as repos and the like.

[4] See also Ayar (2011) for a study on how the U.S. shock affected U.K. banks funding.

However, the tightness of the collateral constraints is not only governed by investors' wealth (Brunnermeier, 2009). Following the financial literature, also changes in counterparty risk affect the tightness of these constraints. This effect is captured in a stylized way by time-varying debt-to-asset ratios. Each borrower faces a unique debt-to-asset ratio which adjusts in accordance with the price of local and foreign collateral in her portfolio (indeed, investors pledge an entire equity portfolio). I first calibrate all the parameters that do not directly influence the debt-to-asset ratios, using in many cases OECD data - coherently with the countries used for Figure 1. Next, the parametrization of the debt-to-asset ratios is the result of an SMM estimation, conditional on the portfolio solution.

Modeled this way, the behaviour of "global funding" can be confronted with the external positions of countries which are subject to negative shocks. For instance, the recent crisis caused negative valuation effects for countries such as the U.S., Japan and the Eurozone; not only asset prices mattered, but also movements in the exchange rate (Gourinchas, Rey and Truempler, 2011). Although my model is real, local and foreign funding can reproduce similar effects. Indeed, the model suggests that an idiosyncratic shock causes a decrease in the net foreign asset position of the country hit by the shock, reverting the predictions of a one-bond-market model and matching U.S. data during the crisis time[5]. This result is justified by the following key results.

The terms of trade (or real exchange rate) risk increases with equilibrium leverage, causing *home funding bias* in equilibrium portfolios. This interaction eliminates any possible effect of leverage on the international diversification of equity holdings. In contrast, absent diversification opportunities in bonds, a one-good model would feature a link between equilibrium leverage and the equilibrium equity portfolio. Consequently, this kind of model would predict that under financial integration the dynamics of leverage are completely absorbed by the world rate of interest, and the collateral constraints would have quantity-effects.

Here the conclusion is different because there are multiple traded bonds. Under financial integration, the terms of trade can open a gap between local and foreign debt. For example, after a negative macroeconomic shock, the economy hit by the shock experiences a real appreciation, so borrowing from foreign financiers is the most severely impaired source of funding. In turn, the real appreciation affects the collateral constraints in each country, so even equity prices react to satisfy general equilibrium.

However, the major effect of wholesale funding on equity prices is represented by how debt-to-asset ratios themselves adjust following the shock. On impact, this adjustment affects equity prices in a negative way, but over the longer run the effect can change sign. This is a sort of "market discipline effect" of haircuts, which I find to be generally stabilizing, except for the case in which macroeconomic shocks are accompanied by some exogenous source of counterparty risk (the type of financial shocks considered herewith).

[5]See Figure 3. below.

The structure of this paper is the following. In section 2., I briefly discuss the literature to which my work is related. Then, I describe the model (section 3.) and how I calibrate it (section 4.). Section 5. is devoted to the study of equilibrium portfolios, while sections 6. and 7. present the corresponding model dynamics. In section 8., I conclude. There are an appendix at the bottom of the paper and a more extensive appendix separate from it.

2 Literature

This paper belongs to the literature on the international transmission of shocks through financial linkages. I build on the portfolio model with leverage constraints of Devereux and Yetman (2010). Other papers on the transmission of shocks through constraints on portfolios are Pavlova and Rigobon (2008) and Dedola and Lombardo (2009). The first paper analyzes the role of the terms of trade in shaping the wealth transfers across countries, but it does not involve financial accelerator effects in financial markets. On the other hand, the second paper embeds a financial accelerator, but it does not adopt collateral constraints and it addresses a different question than mine. Without an endogenous portfolio choice problem, also Kollmann, Enders and Muller (2011) and Van Wincoop (2011) study the international transmission of shocks in two-country models. The first of these two papers is probably closer to the work here, as it analyzes the role of global banks that can receive deposits from local and foreign households. Coherently, banks are modeled as commercial financial institutions which are subject to regulatory capital requirements. In contrast, I consider leveraged institutions that are subject to market-based constraints. Leveraged financial institutions are present also in Van Wincoop's recent framework. He quantifies the transmission under leverage, so some of my results on time-varying debt-to-asset ratios can be related to his findings.

The collateral constraints at work in my model are based on the VaR constraints or total margin constraints used in finance. In this sense, my work is related to Gorton and Pennacchi (1995), who suggest that private guarantees must be risk-sensitive to render marketable otherwise non-marketable assets. Here, equities are accepted as collateral provided that debt-to-asset ratios adjust to counterparty risk. However, my main reference on time-varying margins is Brunnermeier and Pedersen's (2009) study on market liquidity. Other financial papers that influece my analysis are Adrian and Shin (2008), who show how leverage can carefully derived from a binding VaR limit, and Brunnermeier (2009) and Gorton (2009), who describe the role of margins during the 2007-2009 crisis.

In any case, the borrowing limits that I use have a similar form as the one proposed by Kiyotaki and Moore (1997). In this sense, my paper is connected with the studies on the effects of credit constraints on asset prices (Aiyagari and Gertler, 1999), on Sudden Stops in Emerging Economies (Mendoza and Smith, 2006; Mendoza, 2010) and on real estate prices in monetary frameworks (Iacoviello, 2005).

Finally, I benefit from the results obtained by the literature on portfolio choice. I solve the portfolio problem using the method developed by Devereux and Sutherland (2011), and I obtain home bias in both equity and bond holdings. This is possible in my framework as expenditures on goods are characterized by home bias, investment is subject to shocks (Coeurdacier, Kollmann and Martin, 2007; 2010) and the international trade in bonds plays a crucial role (Gourinchas and Coeurdacier, 2009).

3 The Model

Building on Devereux and Yetman (2010), I develop a two-country, two-good, two-agent model with collateral constraints. The two countries are symmetric, and each of them produces a specific good. The tradable portion of this good is produced by firms, which hire labor and accumulate capital over time. The "non-traded" portion of total output is produced (and consumed) in the "backyard" sector. Population is of unit measure and is composed by n impatient households and $1-n$ patient households[6]. Following Adrian and Shin (2009), the impatient households are called "active investors" - thinking of leveraged institutions such as investment banks, big commercial banks and hedge funds - and the patient households are called "passive investors" - referring to mutual funds, insurance companies, pension funds, etc.

Under binding collateral constraints, active investors sell bonds to domestic and foreign passive investors and use the resources so obtained to take long positions in local and foreign firms. Figure 2-A. in the appendix provides a representation of the domestic and international transactions occuring in the model. First, active investors can choose between domestic and a foreign borrowing. Thus, lenders' bond holdings follow implicitly to satisfy equilibrium on debt markets. Second, for a given collateral, total borrowing depends on the debt-to-asset ratio specific to each active investor. Debt-to-asset ratios depend on the price of local and foreign collateral, conditional on the endogenous portfolio choice.

In presenting the model (and in applying it numerically), I shall make comparisons with Devereux and Yetman's framework. The reason for doing this is just to understand more easily how the model below contributes to the crucial transmission mechanism that they formalize. As Figure 3-A. in the appendix shows, in Devereux and Yetman borrowing is modeled as the issuance of a single bond, under two different assumptions: one is the complete segmentation between two domestic markets, the other is the presence of a common worldwide debt market. This latter case is highlighted in bold as it is the one I shall refer to.

3.1 Firms

Goods are differentiated across countries. In each country, the portion of total output which is internationally traded is produced by firms. The objective of these firms is to maximize the present value of

[6] Examples of financial accelerator models where heterogeneous agents discount the future at different rates are Calstrom and Fuerst (1997) and Iacoviello (2005).

future profits:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^S \left(Y_{Ht} - P_t^I I_{t-1} - w_t l \right) \quad ; \quad E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^S \left(Y_{Ft} - P_t^{*I} I_{t-1}^* - w_t^* l^* \right) \tag{1}$$

where the subscript $i = H, F$ refers to a specific good, P_t^I is the price of investment goods, $\Lambda_{0,t}^S$ denotes the stochastic discount factor of shareholders, w_t is the wage rate and l is a fixed amount of labour hours, under the assumption that all agents work for the same amount of time. Variables carrying a "star" refer to the foreign country. Similar notation shall be adopted throughtout all the paper.

Normalizing l to 1, production and capital accumulation can be expressed as follows

$$\begin{aligned} Y_{Ht} &= A_t \left(K_{Ht-1} \right)^{\alpha} 1^{1-\alpha} \quad ; \quad Y_{Ft} = A_t^* \left(K_{Ft-1} \right)^{\alpha} 1^{1-\alpha} & (2) \\ K_{Ht} &= (1-\delta) K_{Ht-1} + \Xi_t I_{t-1} \quad ; \quad K_{Ft} = (1-\delta) K_{Ft-1} + \Xi_t^* I_{t-1}^* & (3) \end{aligned}$$

where A_t, A_t^* are exogenous productivity processes, K_{it} denotes the stock of capital in terms of good i, α is the capital share, δ is the constant rate of depreciation and Ξ_t, Ξ_t^* are investment shocks. These types of shocks are introduced following Greenwood et al. (1997), Fisher (2006) and, for portfolio modeling, Coeurdacier et al. (2010)[7].

Investment expenditures, I_t, I_t^*, are bundles of goods with a bias for the domestic goods:

$$\begin{aligned} I_t &\equiv \left[\gamma_I^{\frac{1}{\theta_I}} \left(I_{Ht} \right)^{\frac{\theta_I - 1}{\theta_I}} + (1-\gamma_I)^{\frac{1}{\theta_I}} \left(I_{Ft} \right)^{\frac{\theta_I - 1}{\theta_I}} \right]^{\frac{\theta_I}{\theta_I - 1}} \quad ; \quad P_t^I = \left[\gamma_I + (1-\gamma_I)\, p_{Ft}^{1-\theta_I} \right]^{\frac{1}{1-\theta_I}} \\ I_t^* &\equiv \left[(1-\gamma_I)^{\frac{1}{\theta_I}} \left(I_{Ht}^* \right)^{\frac{\theta_I - 1}{\theta_I}} + \gamma_I^{\frac{1}{\theta_I}} \left(I_{Ft}^* \right)^{\frac{\theta_I - 1}{\theta_I}} \right]^{\frac{\theta_I}{\theta_I - 1}} \quad ; \quad P_t^{*I} = \left[(1-\gamma_I) + \gamma_I p_{Ft}^{1-\theta_I} \right]^{\frac{1}{1-\theta_I}} \end{aligned} \tag{4}$$

where P_t^I, P_t^{*I} are investment deflators, $\gamma_I > 0.5$ is the share of domestic goods in total investment, and θ_I is the elasticity of substitution between home and foreign goods. Note that, by assumption, the home good is the numeraire and all the other prices are expressed in terms of it. It follows that the *home terms of trade* (ToT) are $1/p_{Ft}$. The law of one price (LOP) holds for each individual good, but due to home bias purchasing power parity (PPP) does not.

Considering production and capital accumulation in the home country as in (2), discounted profits in (1) are maximized when

$$\frac{P_t^I}{\Xi_t} = E_t \Lambda_{t,t+1}^S \left[\alpha A_{t+1} \left(K_{Ht} \right)^{\alpha - 1} + (1-\delta) \frac{P_{t+1}^I}{\Xi_{t+1}} \right]$$

[7] Note that here there is no need to specify the nature of the investment shocks. Ξ_t, Ξ_t^* do not necessarily represent shocks to "investment-specific" technologies, as the most of the literature - included the papers I refer to - generally assumes. Justiniano et al. (2009) show that there is a distinction between the "marginal efficiency" effects of investment and its "investment-specific" component. But this debate is outside the scope of the present paper.

In equilibrium, this condition is satisfied because investment is paid out of retained earnings and shareholders take the resulting dividends as given:

$$d_{Ht} = \alpha \frac{Y_{Ht}}{K_{Ht-1}} - P_t^I \frac{I_{t-1}}{K_{Ht-1}} \tag{5}$$

The wage rate follows as a residual

$$w_t = (1 - \alpha) Y_{Ht} \tag{6}$$

Foreign firms are characterized by an analogous efficiency condition, and foreign factor prices are similar to those in equations (5)-(6).

3.2 Menu of Financial Instruments

There are four internationally traded assets, involving home and foreign equity claims as well as home and foreign bonds.

In each country, the local firms issue claims on productive capital. Only active investors in both countries can purchase equities, so the superscript S in equation (1) refers to both home active investors, A, and foreign active investors, A^*. Consequently, n is the total number of shareholders and the capital stocks are so defined:

$$K_{Ht} = n\chi_{Ht} \quad ; \quad K_{Ft} = n\chi_{Ft} \tag{7}$$

where $\chi_{it} = k_{it}^A + k_{it}^{*A}$ is the *per-capita* amount of traded shares.

The home and foreign bonds are issued by the shareholders themselves, as they seek to increase their financial means beyond their internal resources. Therefore, for active investors bonds are liabilities sold to passive investors in order to borrow their savings. This credit line represents an intra-agents market for funds, through which passive investors take an indirect (long) position in productive firms.

I conveniently express all asset prices and returns in terms of the numeraire (the home good). It follows that q_{it}^e is the i-th equity price, d_{it} is the corresponding dividend payment and q_{it}^b is the price of good i bond. One unit of bond i bought in $t-1$ yields one unit of good i on the following date. So the rates of return on home and foreign equities and the rates of interest on loans are:

$$r_{Ht} = \frac{q_{Ht}^e + d_{Ht}}{q_{Ht-1}^e} \quad ; \quad r_{Ft} = \frac{q_{Ft}^e + d_{Ft}}{q_{Ft-1}^e} \tag{8}$$

$$R_{Ht} = \frac{1}{q_{Ht-1}^b} \quad ; \quad R_{Ft} = \frac{p_{Ft}}{q_{Ft-1}^b} \tag{9}$$

According to this definition, the rate of interest prevailing in each country is riskless *only* for the residents of that country.

3.3 Households

Households in each country are divided in active investors and passive investors. Their common objective is to maximize lifetime utility, which is simply a function of consumption:

$$E_0 \sum_{t=0}^{\infty} \eta_t^h \frac{\left(c_t^h\right)^{1-\sigma}}{1-\sigma} \quad ; \quad E_0 \sum_{t=0}^{\infty} \eta_t^{*h} \frac{\left(c_t^{*h}\right)^{1-\sigma}}{1-\sigma}$$

where $h = A, P$ depending on the household being an active or a passive investor, $1/\sigma$ is the intertemporal elasticity of substitution in consumption and η_t^h is an endogenous discount factor without internalization. As usual, endogeneous discount factors are useful to eliminate the unit root of open economy models[8]; here these discount factors decrease only if the demand for consumption increases on average among the agents of a certain type: $\eta_{t+1}^h = \beta\left(C_t^h\right)\eta_t^h$, where $\beta'\left(C_t^h\right) < 0$ and $\eta_0^h = 1$.

By assumption, passive investors are patient consumers, while active investors are impatient; that is:

$$\beta\left(C_t^P\right) > \beta\left(C_t^A\right) \tag{10}$$

3.3.1 Consumption Demand Functions

In spite of the heterogeneous rate of time preference, active and passive investors allocate equally their consumption expenditures between home and foreign goods. Therefore, home and foreign household h's consumption expenditures are

$$\begin{aligned} c_t^h &\equiv \left[\gamma^{\frac{1}{\theta}}\left(c_{Ht}^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}\left(c_{Ft}^h\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \quad ; \quad P_t = \left[\gamma + (1-\gamma)\, p_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ c_t^{*h} &\equiv \left[(1-\gamma)^{\frac{1}{\theta}}\left(c_{Ht}^{*h}\right)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}\left(c_{Ft}^{*h}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \quad ; \quad P_t^* = \left[(1-\gamma) + \gamma p_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned} \tag{11}$$

where $\gamma > 0.5$ because of the home bias assumption, θ is the elasticity of substitution between the two goods and P_t is the CPI prevailing in the home country. Although consumption and investment bundles are similar, γ, θ are not necessarily equal to γ_I, θ_I, so P_t (P_t^*) may differ from P_t^I (P_t^{*I}). The real exchange rate (RER) is P_t/P_t^*.

3.3.2 Consumption Smoothing: Active Investors

The assumption that active investors are impatient households is made as they are the only traders that operate in international markets; thus, in accordance with Kiyotaki and Moore (1997), they cannot precommit to the full repayment of their debts. Put it another way, active investors have some specific capabilities - in financing capital and diversifying risk - and they can always refuse to support firms[9].

[8] See Schmitt-Grohé and Uribe (2003).

[9] See Hart and Moore (1995).

In such a case, firms would be forced to lower their demand for capital and, as a consequence, current investment and future production.

Since active investors diversify across borders both their capital ownerships and their funding sources, their budget constraint is

$$\begin{aligned} P_t c_t^A - q_{Ht}^b b_{Ht}^A - q_{Ft}^b b_{Ft}^A + q_{Ht}^e k_{Ht}^A + q_{Ft}^e k_{Ft}^A &= w_t - b_{Ht-1}^A - p_{Ft} b_{Ft-1}^A \\ &\quad + \left(q_{Ht}^e + d_{Ht}\right) k_{Ht-1}^A + \left(q_{Ft}^e + d_{Ft}\right) k_{Ft-1}^A \end{aligned} \tag{12}$$

where b_{Ht}^A and b_{Ft}^A denote, respectively, home- and foreign-good bondholdings, and k_{Ht}^A and k_{Ft}^A are home and foreign equityholdings. Foreign active investors' budget constraint is akin to (12). Note that bondholdings carry a negative sign. I keep this convention of the model by Devereux and Yetman (2010), as it makes explicit that active investors take negative (or short) positions in the market for debt securities.

Since active investors cannot promise to repay their debts (and there are no public guarantees in this model), loans must be guaranteed by pledging some collateral. As collateral, active investors use their portfolio of equities. Denoting home and foreign borrowers' debt-to-asset ratios, respectively, as κ_t and κ_t^*, I thus specify the borrowing limits as follows:

$$q_{Ht}^b b_{Ht}^A + q_{Ft}^b b_{Ft}^A \leq \kappa_t \left(q_{Ht}^e k_{Ht}^A + q_{Ft}^e k_{Ft}^A\right) \quad ; \quad q_{Ht}^b b_{Ht}^{*A} + q_{Ft}^b b_{Ft}^{*A} \leq \kappa_t^* \left(q_{Ht}^e k_{Ht}^{*A} + q_{Ft}^e k_{Ft}^{*A}\right) \tag{13}$$

In words, active investors' total sale of debt securities cannot be greater than a fraction of the total assets that they simultaneously purchase[10]. Given the market value of collateral assets, borrowing depends on the maximum debt-to-asset ratio tolerated by financiers. Note that the debt-to-asset ratios are specific to the (residency of the) borrowers and time-varying.

Due to assumption (10) on time preferences, the constraints in (13) will be binding even in the steady state of the model, and active investors will never accumulate so much as to invalidate their borrowing limits (Iacoviello, 2005). So the optimality conditions for home country agents are

$$\left(\lambda_t^A - \mu_t\right) q_{Ht}^b = \beta\left(c_t^A\right) E_t \lambda_{t+1}^A \tag{14}$$

$$\left(\lambda_t^A - \mu_t\right) q_{Ft}^b = \beta\left(c_t^A\right) E_t \lambda_{t+1}^A p_{Ft+1} \tag{15}$$

$$\left(\lambda_t^A - \mu_t \kappa_t\right) q_{Ht}^e = \beta\left(c_t^A\right) E_t \lambda_{t+1}^A \left(q_{Ht+1}^e + d_{Ht+1}\right) \tag{16}$$

$$\left(\lambda_t^A - \mu_t \kappa_t\right) q_{Ft}^e = \beta\left(c_t^A\right) E_t \lambda_{t+1}^A \left(q_{Ft+1}^e + d_{Ft+1}\right) \tag{17}$$

where $\lambda_t^h = \left(c_t^h\right)^{-\sigma}/P_t$ is agent h's marginal utility of consumption and μ_t the marginal utility of borrowing under collateral. Similar conditions are derived for foreign active investors.

It is then easy to see from (14)-(15) that the ratio μ_t/λ_t^A constitutes the main part of the premium that active investors must pay for their impatience, regardless of the type of bond they sell in order to borrow.

[10] See Gorton and Pennacchi (1990; 1995).

3.3.3 Consumption Smoothing: Passive Investors

Patient houholds are "passive" investors because they take only *indirect* positions in domestic investment and international financial markets. Patient investors save goods to finance active investors' capital ownership, and in the international capital market active investors sell local equities for foreign equities. At the same time, active investors give away the local bonds that domestic lenders agree to buy from foreign borrowers and receives the foreign bonds that foreign lenders agree to buy from home borrowers. In other words, passive investors do not face any real exchange rate risk, which means that they can *only* lend in terms of the domestic good and the cross-border diversification of funding is endogenous to active investors' choice between home and foreign good loans.

Since only active investors face a choice between domestic and foreign loans, passive investors grant a loan whose total size depend on the total amount of bonds issued in a given good by home and foreign borrowers:

$$B^P_{Ht} \equiv b^P_{Ht} + b^{*P}_{Ht} \quad ; \quad B^P_{Ft} \equiv b^P_{Ft} + b^{*P}_{Ft}$$

Aside from their lending activity, passive investors carry out a residual production process. Running a "backyard" technology, passive investors absorb a portion of the total stock of capital available in the economy. Let home and foreign financiers' production functions, respectively, be $z\left(k^P_{Ht-1}\right)^{\nu}$ and $z\left(k^{*P}_{Ft-1}\right)^{\nu}$, where $\nu < 1$. So, while firms produce at constant returns to scale (equation (2)), passive investors' production features decreasing returns to scale. And since k^P_{Ht}, k^{*P}_{Ft} are not marketed, passive investors' capital holdings do not contribute to capital accumulation in the overall economy. Thus, aggregate capital accumulation is solely given by equation (3). In addition, the productivity z is fixed by assumption, and the product realized in the backyard sector is a particular case of non-traded good. By assumption, the output of this sector is totally consumed inside the sector itself, meaning that the only consumer of this output is the passive investor who produces it.

Summing up, passive investors' budget constraints are

$$P_t c^P_t + q^e_{Ht}\left(k^P_{Ht} - k^P_{Ht-1}\right) - q^b_{Ht} B^P_{Ht} = w_t + z\left(k^P_{Ht-1}\right)^{\nu} - B^P_{Ht-1} \tag{18}$$

$$P^*_t c^{*P}_t + q^e_{Ft}\left(k^{*P}_{Ft} - k^{*P}_{Ft-1}\right) - q^b_{Ft} B^P_{Ft} = w^*_t + p_{Ft} z\left(k^{*P}_{Ft-1}\right)^{\nu} - p_{Ft} B^P_{Ft-1} \tag{19}$$

Maximizing passive investors lifetime utility under these constraints yields

$$\lambda^P_t q^b_{Ht} = \beta\left(c^P_t\right) E_t \lambda^P_{t+1} \tag{20}$$

$$\lambda^P_t q^e_{Ht} = \beta\left(c^P_t\right) E_t \lambda^P_{t+1}\left[q^e_{Ht+1} + \nu z\left(k^P_{Ht}\right)^{\nu-1}\right] \tag{21}$$

for the home country, and

$$\lambda^{*P}_t q^b_{Ft} = \beta\left(c^{*P}_t\right) E_t \lambda^{*P}_{t+1} p_{Ft+1} \tag{22}$$

$$\lambda^{*P}_t q^e_{Ft} = \beta\left(c^{*P}_t\right) E_t \lambda^{*P}_{t+1}\left[q^e_{Ft+1} + p_{Ft} \nu z\left(k^{*P}_{Ht}\right)^{\nu-1}\right] \tag{23}$$

for the foreign country.

3.4 Competitive Equilibrium

The clearing conditions on the markets for goods, bonds and equities are, respectively, as follows:

$$n\left(c_{Ht}^{A}+c_{Ht}^{*A}\right)+I_{Ht}+I_{Ht}^{*}+(1-n)\left(c_{Ht}^{P}+c_{Ht}^{*P}\right) = Y_{Ht}+(1-n)\,z\left(k_{Ht-1}^{P}\right)^{\nu} \quad (24)$$

$$n\left(c_{Ft}^{A}+c_{Ft}^{*A}\right)+I_{Ft}+I_{Ft}^{*}+(1-n)\left(c_{Ft}^{P}+c_{Ft}^{*P}\right) = p_{Ft}\left[Y_{Ft}+(1-n)\,z\left(k_{Ft-1}^{*P}\right)^{\nu}\right] \quad (25)$$

$$n\left(b_{Ht}^{A}+b_{Ht}^{*A}\right)+(1-n)\,B_{Ht}^{P}=0 \quad ; \quad n\left(b_{Ft}^{A}+b_{Ft}^{*A}\right)+(1-n)\,B_{Ft}^{P}=0 \quad (26)$$

$$n\chi_{Ht}+(1-n)\,k_{Ht}^{P}=1 \quad ; \quad n\chi_{Ft}+(1-n)\,k_{Ft}^{*P}=1 \quad (27)$$

The terms on the left hand side of (24)-(25) are consumption and investment demand functions across agents and countries: these expenditures are allocated between home and foreign goods as implied by equations (4) and (12).

Therefore, for $t=0,...,\infty$, the competitive equilibrium consists of a vector of allocations ($c_{H,t}^{A}$, $c_{F,t}^{A}$, $c_{H,t}^{*A}$, $c_{F,t}^{*A}$, $c_{H,t}^{P}$, $c_{F,t}^{P}$, $c_{H,t}^{*P}$, $c_{F,t}^{*P}$, I_{Ht}, I_{Ft}, I_{Ht}^{*}, I_{Ft}^{*}, b_{Ht}^{A}, b_{Ht}^{*A}, b_{Ft}^{A}, b_{Ft}^{*A}, B_{Ht}^{P}, B_{Ft}^{P}, k_{Ht}^{A},k_{Ht}^{*A}, k_{Ht}^{P}, k_{Ft}^{A}, k_{Ft}^{*A}, k_{Ft}^{*P}) and of a vector of prices (P_t, P_t^*, P_t^I, P_t^{*I}, p_{Ft}, q_{Ht}^{e}, q_{Ft}^{e}, q_{Ht}^{b}, q_{Ft}^{b}, w_t, w_t^*, d_{Ht}, d_{Ft}) such that: a) firms in both countries maximize profits; b) active investors in both countries maximize lifetime utility subject to their budget and collateral constraints; c) passive investors in both countries maximize lifetime utility subject to their budget constraints; d) the six markets all clear. The full list of equilibrium conditions can be found in the separate technical appendix.

3.5 Comparing Borrowing Limits and Modeling Debt-to-Asset Ratios

Devereux and Yetman (2010) build a one-good model and specify the credit constraints as leverage constraints. In this case, home active investors are subject to

$$b_t^A \leq \kappa\left(q_{Ht}^{e}k_{Ht}^{A}+q_{Ft}^{e}k_{Ft}^{A}\right) \quad (28)$$

where b_t^A is expressed in terms of the unique good and κ is constant. Under bond market integration, home active investors' borrowing is thus a *given* share of the savings granted by all passive investors; the remaining part goes to foreign borrowers. In fact, in that version of Devereux and Yetman's model there is a unique bond market clearing condition, which is

$$n\left(b_t^{A}+b_t^{*A}\right)+(1-n)\left(b_t^{P}+b_t^{*P}\right)=0 \quad (29)$$

In comparison to Devereux and Yetman, here the cross-border diversification between goods implies the presence of two sources of funding. With (13) replacing (28), home active investors can issue both home-good bonds, b_{Ht}^{A}, and foreign-good bonds, b_{Ft}^{A}. Instead of (29), the model can be closed with the two clearing conditions in equation (26). These two segments of the world market for debt securities are clearly integrated, but *de facto* the share of home versus foreign borrowing depends on the fluctuations of the ToT (and the RER). In fact, both budget constraints and borrowing limits are affected by the prices of the bonds, q^b_{Ht} and q^b_{Ft} (equations (12), (13)), and the factor distinguishing passive investors' Euler conditions for bonds is p_{Ft} (equations (20), (22)).

First, the presence of q^b_{Ht} and q^b_{Ft} in the left hand side of (13) means that international financial integration affects the collateral constraints. One formal manipulation of that equation that shows this aspect for home borrowers yields

$$b^A_{Ht} + R_{Ht+1} q^b_{Ft} b^A_{Ft} \leq R_{Ht+1} \left[\kappa_t \left(q^e_{Ht} k^A_{Ht} + q^e_{Ft} k^A_{Ft} \right) \right] \tag{30}$$

$$R_{Ft+1} q^b_{Ht} b^A_{Ht} + p_{Ft+1} b^A_{Ft} \leq R_{Ft+1} \left[\kappa_t \left(q^e_{Ht} k^A_{Ht} + q^e_{Ft} k^A_{Ft} \right) \right] \tag{31}$$

The main features of these two equations are: the ToT $1/p_{Ft}$, its interaction with agents' discount factors, and the link between loan rates and κ_t.

This link is present by construction in models that employ ex post collateral constraints (e.g., Kiyotaki and Moore, 1997; Iacoviello, 2005), whereas it is (*formally*) absent in many models that employ ex ante collateral constraints (e.g., Aiyagari and Gertler, 1999; Mendoza and Smith, 2006). The main difference between these two types of models is that, while ex ante constraints restraint the flow of interest payments on debt, ex post constraints impose a limit on the amount of borrowing. Both the leverage constraint (28) and the collateral constraints (13) assume the ex-ante-like form. But a simple comparison shows that the leverage constraint (28) does not involve any formal relation between interest rate and debt-to-asset ratio because, by assumption, all investors borrow issuing the same type of bond on a unique market. On the other hand, in this model the global bond market has both a domestic and a foreign segment. As a consequence, bond prices differ across borders, and there is an implicit interaction between κ_t, on one side, and R_{Ht} and R_{Ft}, on the other (equations (30), (31)).

Second, the liability portfolio can be solved *endogenously* because, according to (13), each borrower is subject to a unique constraint that involves both bonds and because the constraint is linear. The model by Iacoviello and Minetti (2006) represents an alternative approach, where the choice between cross-border funding is somewhat exogenous. In that model, each borrower faces two collateral constraints and foreign lenders pay higher costs than local lenders to seize collateral in case of default. Thus, the constraint is non-linear, and home and foreign borrowing depend on the share of the collateral pledged to home and foreign lenders.

Finally, debt-to-asset ratios are not constant as in (28) but time-varying. As shown in appendix A.1, the collateral constraints in (13) imply the margin (or haircut) constraints used in the finance literature, which has recently shown that debt margins are set on the basis of a time-dependent distribution of losses. By definition, these margins equal the difference between the market value of pledged collateral and the size of the loan and are generally specific to a given collateral asset. Moreover, they can be seen as a *comprehensive* measure of counterparty risk in collateralized borrowing, as haircuts capture both borrowers' creditworthiness and the risk of the collateral asset (Gorton, 2009).

I thus model the debt-to-asset ratios incorporating these results from the finance literature in my framework. Following Brunnermeier and Pedersen (2009), I define debt-to-asset ratios specific to home and foreign colleteral as

$$\kappa_{it} = f\left(\frac{q^e_{it}}{q^e_{it-1}}\right) \text{ with } i = H, F \ , \ f'(\cdot) > 0 \tag{32}$$

When collateral constraints bind, lenders adjust their savings supply accordingly with the change in equity prices (see appendix A.1). Assume for example that there is an idiosyncratic increase in the market value of home equities. Other things being equal, home collateral is more valuable now than before the shock, so κ_{Ht} must rise reflecting the increased willingness of financiers to lend against home equities. But equation (13) does not capture this sort of events, because loan contracts impose a unique debt-to-asset ratio on home borrowers; this ratio depends on the total value of their portfolios rather than the single elements composing it. So what matters here for the behaviour of κ_t is the endogenous portfolio choice (as opposed to other factors that more explcitly affect agents' creditworthiness). Formally:

$$\kappa_t = \frac{q^e_{Ht}k^A_{Ht}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}}\kappa_{Ht} + \frac{q^e_{Ft}k^A_{Ft}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}}\kappa_{Ft} \tag{33}$$

where $q^e_{Ht}k^A_{Ht}/\left(q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}\right)$ and $q^e_{Ft}k^A_{Ft}/\left(q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}\right)$ are portfolio shares of home and foreign equities, respectively. Then, using (32) I obtain a linear debt-to-asset setting equation, which is empirically applicable:

$$\kappa_t = \psi\frac{q^e_{Ht}}{q^e_{Ht-1}} + \psi^*\frac{q^e_{Ft}}{q^e_{Ft-1}} + \frac{\bar{m}}{\bar{\kappa}}\epsilon_{\kappa t} \tag{34}$$

where, for some function $g[\cdot]$, $\psi = g\left[q^e_{Ht}k^A_{Ht}/\left(q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}\right)\right]$, $\psi^* = g\left[q^e_{Ft}k^A_{Ft}/\left(q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}\right)\right]$, $\bar{m}/\bar{\kappa}$ is a scaling factor[11] (with $\bar{m} = 1-\bar{\kappa}$ being th margin) and $\epsilon_{\kappa t}$ is an exogenous innovation to κ_t. Since the model at hand is a symmetric framework, under portfolio choice also κ^*_t is symmetrically defined[12]. In other words, ψ, ψ^* govern the fluctuations of debt-to-asset ratios symmetrically across countries. To solve for portfolios endogenously in presence of (31) and its foreign counterpart, I estimate ψ by simulated method of moments (SMM), with ψ being conditional on the portfolios endogenously chosen. Finally, I derive ψ^* as well. This estimation approach is explained below.

The scaling factor equals the inverse of borrowers' debt-to-wealth ratio in the steady state. Intuitively, inspired by Adrian and Shin (2008), I introduce it to capture borrowers' initial leverage when an exogenous shock spurs the reaction of model variables. Technically, $\bar{m}/\bar{\kappa}$ transforms the size of $\epsilon_{\kappa t}$ in such a way that a unit shock to κ_t gives rise to a unit shock to haircuts.

[11] Variables surmounted by a "bar" denote steady state values.

[12] Foreign active investors' debt-to-asset ratio is

$$\kappa^*_t = \psi^*\frac{q^e_{Ht}}{q^e_{Ht-1}} + \psi\frac{q^e_{Ft}}{q^e_{Ft-1}} + \frac{\bar{m}}{\bar{\kappa}}\epsilon_{\kappa^* t}$$

4 Calibration

The stylized margin setting behaviour considered in this paper depends on the value of equities through parameters that are estimated in a structural way. In this section I describe my approach to perform this computation separately from the more standard calibration of all the other model parameters.

4.1 Parameters Not Governing Borrowers' Margins

The calibration of parameters other that the debt-to-asset ratio is based on previous studies as well as on data. The behavioural parameters are from previous studies. I use U.K. and U.S. data to calibrate the borrowing costs, because these two countries are the major international financial centres. The parameters governing the exogenous states are instead computed over a sample of OECD countries. This sample is formed by the G10 countries, Australia and Switzerland[13]; thus, the sample involves a quite ample range of advanced economies, largely matching the countries which are considered in Figure 1 on cross border banking. In any case, I simply use yearly data, and coherently all the model is calibrated in years.

Starting with the behavioural parameters, I follow Devereux and Yetman (2010) for those parameters that are present also in their model. This is the case of the specification for the households' discount factor, which is $\beta\left(c_t^h\right) = \zeta^h\left(1+c_t^h\right)^{-\phi}$ with $h = A, P$, and of the values for n, ϕ, σ, z and ν. In contrast, parameters such as γ, θ, γ_I, θ_I are not present in their model. The values chosen for γ and θ are from Corsetti, Dedola and Leduc (2008), who consider OECD countries and show that the elasticity of substitution between traded goods θ is below 1 (i.e., 0.85). I calibrate γ_I, θ_I similarly, but I set them slightly above γ, θ. In this way, in the steady state equilibrium $\bar{P}$ differs from $\bar{P}^I$; this difference plays a role in the calibration of the investment shock, which is described below.

The borrowing costs are based on the 3-month U.K. and U.S. LIBORs in the period from December 31, 1998 to September 16, 2009 and on the 3-month overnight interest swap rate (OIS) recently observed on the U.S. market. I calibrate the (gross) interest rate converting the LIBOR data on a yearly basis and averaging across both dates and markets. As a result, in the steady state $\bar{R} = 1/\beta\left(\bar{c}^P\right) = 1.0418$, where the second equality is from (20) and (22). On the other hand, since the collateral constraints bind even in the steady state, the active investor must pay an extra-cost. Given (14)-(15) and their foreign counterparts, worldwide equilibrium requires that

$$\frac{1}{\beta\left(\bar{c}^A\right)} - \frac{\bar{\mu}}{\beta\left(\bar{c}^A\right)\bar{\lambda}^A} = \bar{R}\left(= \frac{1}{\beta\left(\bar{c}^P\right)}\right)$$

which means that $\bar{\mu}/\beta\left(\bar{c}^A\right)\bar{\lambda}^A$ is the loan premium paid by the active investor (defined as *guarantee premium*, see section 6). I calibrate this premium using the 3-month LIB-OIS spread, which is considered

[13]The countries in the sample are Australia, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, the U.K. and the U.S.

to be a good metric for the state of the U.S. interbank market (Gorton and Metrick, 2009b). To avoid the effect of the recent financial crisis on this spread, I consider observations from January 2004 to August 2007 and find that the average spread for this period equals 11.34 basis points[14].

The parameters governing the exogenous state variables are calibrated using the OECD data (years 1970-2010). In the model, there are three forcing variables, but one of them is the shock to the debt-to-value ratio. Leaving this for the next section, the remaining forcing variables are the productivity shock and the investment shock, which are specified as follows:

$$\begin{aligned}\ln A_t &= \rho_A \ln A_{t-1} + \varepsilon_{At} \text{ with } \varepsilon_{At} \sim N\left(0, \sigma_A^2\right) \\ \ln \Xi_t &= \rho_\Xi \ln \Xi_{t-1} + \varepsilon_{\Xi t} \text{ with } \varepsilon_{\Xi t} \sim N\left(0, \sigma_\Xi^2\right)\end{aligned}$$

My general assumption is that shocks of the same nature can be correlated across countries, while shocks of different nature are uncorrelated between them. For instance, $\rho\left(\varepsilon_{At}, \varepsilon_{\Xi t}\right) = 0$ but $\rho\left(\varepsilon_{At}, \varepsilon_{A^* t}\right) \neq 0$.

For the productivity shock, I derive Solow Residuals. I build these residuals using data on GDP, employment and capital. For the investment shock, I follow Fisher (2006) and use the ratio P_t/P_t^I, which is empirically equal to the ratio between CPI and investment deflator[15]. In performing all these calculations, I prefer to omit Germany due to missing observations, and the construction of Solow Residuals required to leave out Switzerland as well. With this adjustment, I obtain persistence, volatility and cross-country correlations for both shocks, as reported in the last part of Table 1[16]. The computation of these values is standard. For each country, I fit an AR(1) process on logarithmic and linearly detrended variables. I then obtain cross-country correlations and cross-country averages.

4.2 Parameters Governing Borrowers' Margins

To parametrize the debt-to-asset ratio setting equations (equation (34) and its foreign counterpart), I use OECD data once again. The countries in the sample are the same as those used for the other parameters. The parametrization of equation (34) involves the steady state ratio, $\bar{\kappa}$, the parameters ψ, ψ^* that govern the dynamics and the specification for the shock $\epsilon_{\kappa t}$. In the steady state $\bar{\kappa} = \psi + \psi^*$, so I adopt a mixed approach: $\bar{\kappa}$ is computed using observed statistics on financial institutions, ψ is estimated upon an assumed specification for $\epsilon_{\kappa t}$ and, finally, ψ^* is computed as a residual. The justification for this approach is that I treat the debt-to-asset ratio as an unobservable variable. Data on the value of this ratio in the international interbank market are scarce, although there has been some attempts to proxy it (Brunnermeier and Pedersen, 2009; Gorton and Metrick, 2009a,b).

[14] During the crisis, the same spread reached levels even higher than 1 to 2 percentage points.

[15] As investment deflator, I take the gross total fixed capital formation deflator.

[16] The cross-country correlation between Solow Residuals might seem very high, yet it is what I obtain when I extend my sample. Early results based only on Canada, Japan, the U.K. and the U.S. showed a much smaller correlation. I can thus conclude that the high value obtained with the second sample is due to the inclusion of the European countries.

The steady state debt-to-asset ratio $\bar{\kappa}$ is determined matching the share of equities in total assets of the institutional investors operating in the OECD countries of my sample. In fact, the level of leverage achieved by cross-border financial intermediaries must be consistent with both the liability side of their balance sheets and the asset side of their lenders' balance sheets. Interpreting financiers as passive investors (Adrian and Shin, 2009), I use the OECD statistics on institutional investors. The two categories I take into account are "investment funds, consolidated" (i.e., mutual funds) and "insurance corporations and pension funds, consolidated". Considering data from 1999 to 2006[17], I thus match the ratio $\bar{q}^e\bar{k}^P / \left(\bar{q}^e\bar{k}^P - \bar{q}^b\bar{B}^P\right)$ with the empirical share of equities in total assets. As shown in Table 2, averaging across countries and time, I obtain a ratio of 0.49 for the investment funds and one of 0.42 for the insurance companies and pension funds. So I set $\bar{q}^e\bar{k}^P / \left(\bar{q}^e\bar{k}^P - \bar{q}^b\bar{B}^P\right) = 0.45$, rendering $\bar{\kappa} = 0.31$; that is, a steady state leverage of 1.45.

$\bar{\kappa} = 0.31$ might seem a bit low debt-to-asset ratio, and a caveat of my computation is that I am implicitly assuming that all the assets that institutional investors do not hold in the form of equities are lent to leveraged investors in the interbank market. However, the frequency of the data is quite low and my calibration is fairly in line with the evidence on the (liabilities of the) U.S. shadow banking system. First, I am considering average figures computed on yearly observations. Hence, my computation amounts to asking how many funds active investors can on average obtain against their own assets within an entire year. In contrast, the frequency of the typical collateralized loan granted on the interbank market (e.g., the overnight deposit) is much higher, so it is clear that such a loan features a much higher debt-to-asset ratios. Second, as the second panel of Table 2 shows, Federal Flow of Funds data on American shadow banks imply a similar value for $\bar{\kappa}$ which is about the same as the one derived from the OECD data. In fact, shadow banks are leveraged financial institutions, which correspond to the active investors in the model. Over a comparable period of time, I find that for these banks $\bar{\kappa}$ is between 0.35 and 0.29[18]. The latter ratio refers to the case where household and public sectors are eliminated from the credit market borrowing of banks (second column). This case is consistent with my focus on wholesale funding.

Knowing $\bar{\kappa}$, ψ and ψ^* are estimated by SMM, which is a suitable procedure to use when a variable - or a proxy for it - is not available in observed datasets (Duffie and Singleton, 1993). Actually, since $\psi^* = \bar{\kappa} - \psi$, the estimation requires the *identification* of just one parameter, which turns out to be ψ (see Figure 1. in the separate technical appendix). Equation (34) is derived in accordance with the finance literature on VaR constraints, in which case, the volatility of the returns on the collateral asset follow a GARCH process. So under binding VaR constraints also the implied debt-to-asset ratios display GARCH-like movements. I proceed in two steps. First, I proxy the shock $\epsilon_{\kappa t}$ with the (log-)return on

[17] Including the observations for 2007 might affect the computation of a steady state parameter through the effects of the crisis (i.e., financiers hoarding behaviour), so I remove that observation. However, the averages over the period 1999-2007 are almost identical to those reported in Table 2.

[18] I define the shadow banking sector as indicated at the bottom of Table 2.

equities and assume, for simplicity, that it follows a standard macroeconomic specification. Second, since such a specification amounts to an approximation of the financial approach, I choose some of my moments to control for effects that the locally approximated solution used here cannot reproduce (the solution method is in the next section).

My assumption for $\epsilon_{\kappa t}$ is simply that it is akin to the other shocks in the model:

$$\epsilon_{\kappa t} = \rho_{\varkappa} \epsilon_{\kappa t-1} + \varepsilon_{\kappa t} \text{ with } \varepsilon_{\kappa t} \sim N\left(0, \sigma_{\kappa}^2\right) \tag{35}$$

The set of empirical moments that I match is made of the following four moments: 1) the consumption-output correlation; 2) the correlation between the stock price and output; 3) the skewness of equity returns; 4) the kurtosis of equity returns. The kurtosis and skewness of equity returns are the moments chosen to control for GARCH-like effects. An additional justification for this choice is that kurtosis and skewness increase the performance of SMM methods for models with RE pricing equations (Michaelides and Ng, 2000). In accordance with the selected moments, from the OECD database I extract data on output, consumption and the stock market index[19]. In terms of the model, output is a proxy for Y_{Ht}, consumption is coherently the total consumption of traded goods produced in the home country ($c_t = n\left(c_{Ht}^A + c_{Ft}^A\right) + (1-n)\left[c_{Ht}^P + c_{Ft}^P - z\left(k_{Ht-1}^P\right)^{\nu}\right]$) and the stock market index is q_{Ht}^e. The range of the observations that I use is 1975-2010; this means that $T = 35$ as the empirical returns on equities are $\ln r_t = \ln q_t^e - \ln q_{t-1}^e$. Table 3. reports the results obtained fitting equation (35) with $\ln r_t$ proxying for $\epsilon_{\kappa t}$ and estimating ψ by SMM on the basis of ρ_κ, σ_κ, $\rho\left(\varepsilon_{\kappa t}, \varepsilon_{\kappa^* t}\right)$ so obtained - plus clearly all the above calibration.

The parameters concerning $\epsilon_{\kappa t}$ show that shocks to debt-to-value ratios are less persistent than the other macroeconomic shocks, but their second moments are much higher. One would in fact expect financiers to react strongly to exogenous shocks to borrowers' counterparty risk (a source of uncertainty on the true ability of borrowers to repay) and, simultaneously, to revise quickly their expectations. On the other hand, $\rho\left(\varepsilon_{\kappa t}, \varepsilon_{\kappa^* t}\right) = 0.64$ is a quite big number, which mainly depends on the high cross-country correlation between stock market returns - and, thus, the variable chosen as a proxy for $\epsilon_{\kappa t}$.

The SMM estimation is run using three different lengths for the time-series generated by the model, which is $T_s = NT$. Setting N equal to 10, 20 and 30, I simulate time-series of 350, 700 and 1050 data-points. In order to control for small sample bias, I repeat the simulation N times instead of extracting a unique simulated sample of NT observations. All the other details on the SMM procedure effectively used can be found in the separate technical appendix.

As Table 3. shows, the estimated value for ψ is between 0.12 and 0.13 for any T_s. However, since the asymptotic properties of the estimator improve as $N \to \infty$, I take $\psi = 0.126$ (the value obtained with

[19]Specifically, the empirical time-series I consider are: GDP at current market prices, private final consumption expenditures and "share prices, index". As before, to account for missing values, I shrink my sample eliminating Germany and, additionally, Belgium.

$N = 30$) as my most reliable estimation. It follows that $\psi^* = 0.185$. The last four columns of Table 3 show how good this estimation is in terms of matching empirical moments with theoretical moments. Given the fact that the model does not contain many of the features typical of an RBC framework, it is reasonable to expect that the empirical moments will not perfectly matched by the theoretical ones. For instance, $\rho\left(c_t Y_{Ht}\right)$ is roughly 0.2 higher in the data than in the model, but the model does not allow for habits in consumption, labor in output and utility, and so on. The correlation between the price of equities and output is almost perfectly matched, and so is remarkably the matching of the kurtosis[20]. In contrast, the model finds it difficult to match the empirical skewness of equity returns, although the sign is correct as $N \rightarrow \infty$ (i.e., for $N = 20, 30$). One explanation for this weakness of the model is that I am using local solution methods for macro-models, so a negative empirical skewness of 0.7 is (in absolute terms) too high to reproduce with quantitative simulations.

Nevertheless, this weakness is not so large to impair the estimation. Since I match four moments to estimate just one parameter, there are three overidentification restrictions (OIR) to test. And, at 5 percent significance, this test cannot reject the hypothesis that the three extra-moments are valid to identify ψ.

5 Portfolio Choice and Solution

5.1 Choice Problem with a Collateralized Liability Portfolio

The heterogeneity between agents and the structure of international financial markets are such that only active investors face a portfolio choice problem, which involves both equities and bonds[21]. From this viewpoint, bonds are seen as financial instruments sold to passive investors in order to diversify borrowing across borders. In this section, I analyze this choice between home and foreign liabilities, showing how the international portfolio problem can be set up under binding collateral constraints.

Given the consumption Euler equations (14)-(17), home agents' choice problem is solved by the following portfolio Euler equations:

$$E_t \lambda_{t+1}^A \left(R_{Ht+1} - R_{Ft+1}\right) = 0 \quad (36)$$

$$E_t \lambda_{t+1}^A \left(r_{Ht+1} - r_{Ft+1}\right) = 0 \quad (37)$$

Foreign active investors face an equivalent portfolio problem. Note that the portfolio selection equations abstract from any sorts of comparisons between bond and equity claims. This is a specific feature of choosing home and foreign liabilities under binding collateral, as explained in Remarks 1 and 2 below.

[20] The return data used here turn out to be slightly platykurtic, which is probably due to the fact that they are computed from yearly observations on stock prices. Fatter tails show up at higher frequencies.

[21] See Figure 1-A. in appendix A.3.

Adopting the solution method developed by Devereux and Sutherland (2011), the equilibrium portfolios are computed taking a second order approximation of the portfolio Euler equations across countries and writing the model in such a way that it shows the effect of portfolios on investors' budget constraints. Only the budget constraint of home active investors is taken into account.

Writing in compact form and using the definition of λ_t^A, the (cross-border) portfolio Euler is

$$\left[E_t\left(c_{t+1}^A\right)^{-\sigma}\frac{1}{\pi_{t+1}}-E_t\left(c_{t+1}^{*A}\right)^{-\sigma}\frac{1}{\pi_{t+1}^*}\right]\left[\begin{array}{c}R_{xt+1}\\ r_{xt+1}\end{array}\right]=0 \tag{38}$$

where $\pi_t = P_t/P_{t-1}$ ($\pi_t = P_t^*/P_{t-1}^*$), $R_{xt} = R_{Ht} - R_{Ft}$ is the "interest rate differential" and $r_{xt} = r_{Ht} - r_{Ft}$ is the excess returns on home versus foreign equities. R_{xt} affects the bond portfolio, $\omega_t^b = q_{Ht}^b\left(b_{Ht}^A - B_{Ht}^A\right)$ (with $B_{it}^A = b_{it}^A + b_{it}^{*A}$, $i = H, F$), while r_{xt} influences the equity portfolio, $\omega_t^e = q_{Ht}^e\left(k_{Ht}^A - \chi_{Ht}\right)$. These portfolios show up in home agents' budget constraint, which takes the following form:

$$\begin{aligned}P_t c_t^A + NFE_t^A - NFB_t^A &= w_t - q_{Ht}^e\left(\chi_{Ht}-\chi_{Ht-1}\right)+d_{Ht}\chi_{Ht-1}+q_{Ht}^b B_{Ht}^A - B_{Ht-1}^A\\ &\quad + r_{xt}\omega_{t-1}^e - R_{xt}\omega_{t-1}^b + r_{Ft}NFE_{t-1}^A - R_F NFB_{t-1}^A \end{aligned}\tag{12'}$$

where $NFE_t^A = q_{Ft}^e k_{Ft}^A - q_{Ht}^e\left(\chi_{Ht} - k_{Ht}^A\right)$ is the net foreign equity position and $NFB_t^A = q_{Ft}^b b_{Ft}^A - q_{Ht}^b\left(B_{Ht}^A - b_{Ht}^A\right)$ is the net foreign bond position. Following Devereux and Yetman (2010), $\omega_{t-1}^j < 0$ means that home active investors hold less than 100 percent of the domestic per-capita stock of instrument j, with $j = e, b$. Equation (12') shows simply that the dynamics of NFE_t net of NFB_t depend on capital and non-capital income (net of consumption expenditures). Capital income has various components. To start with, capital income is bigger, the bigger the difference between the returns earned on the foreign equities from $t-1$ and the interests paid on foreign liabilities (last two terms). Other sources of income are dividend payments, $d_{Ht}\chi_{Ht-1}^A$, and the excess returns of home versus foreign equities, $r_{xt}\omega_{t-1}^e$, while interest payments to local financiers, B_{Ht-1}^A, and the excess cost of home versus foreign funds, $R_{xt}\omega_{t-1}^b$ have the opposite effect of reducing wealth. Finally, new resources come from the sale of new bonds, $q_{Ht}^b B_{Ht}^A$, but increasing the capital stock available in the traded sector respresents a cost, $q_{Ht}^e\left(\chi_{Ht} - \chi_{Ht-1}\right)$.

As the budget constraint, also the collateral constraints (13) and the market clearing conditions (26) must be coherently expressed in terms of the new variables. For example, in case of home active investors, one obtains

$$NFB_t^A + q_{Ht}^b B_{Ht}^A \leq \kappa_t\left(NFE_t^A + q_{Ht}^e\chi_{Ht}\right) \tag{13'}$$

$$nB_{Ht}^A + (1-n)B_{Ht}^P = 0 \tag{26'}$$

In sum, the model is made of all the conditions that define the equilibrium, provided that the constraints and the bond market clearing conditions are transformed as shown in (12'), (13') and (26'). The model is solved with a first order approximation, which is then used to satisfy a second order approximation of the portfolio equations, combined as in (38). These steps are useful for the estimation of ψ, which is then conditional on the portfolios, as well as for solving and simulating the model once the optimal ψ has been found (further details are in the separate appendix).

Remark 1. Due to the collateral constraint, the comparison between assets belonging to the collateralized portion of portfolios and those used as collateral is redundant. Hence, the international portfolio problem has a smaller dimension than the one it would have under non-binding collateral constraints.

Consider applying the usual approach to portfolio choice straightforwardly, neglecting the fact that collateral constraints are binding in both countries. Let the foreign equity be the reference asset, so that equations (14)-(17) yield

$$\beta\left(c_t^A\right) E_t \lambda_{t+1}^A \left(R_{Ht+1} - r_{Ft+1}\right) + \mu_t \left(1 - \kappa_t\right) = 0 \quad (39)$$

$$\beta\left(c_t^A\right) E_t \lambda_{t+1}^A \left(R_{Ft+1} - r_{Ft+1}\right) + \mu_t \left(1 - \kappa_t\right) = 0 \quad (40)$$

$$E_t \lambda_{t+1}^A \left(r_{Ht+1} - r_{Ft+1}\right) = 0$$

where each of the first two equations is a no-arbitrage condition between a bond and an equity. This sort of condition features a term, $\mu_t \left(1 - \kappa_t\right)$, which is not generally treated by the existing solution methods for international portfolios. And this could represent a potential problem.

Coherently with the three conditions obtained, one would need to determine three portfolio shares. So defining these three shares as $\omega_{Ht}^b = q_{Ht}^b \left(b_{Ht}^A - B_{Ht}^A\right)$, $\omega_{Ft}^b = q_{Ft}^b b_{Ft}^A$, $\omega_t^e = q_{Ht}^e \left(k_{Ht}^A - \chi_{Ht}\right)$ and the home investors' net foreign assets as

$$NFA_t^A = q_{Ft}^e k_{Ft}^A - q_{Ft}^b b_{Ft}^A - q_{Ht}^e \left(\chi_{Ht} - k_{Ht}^A\right) + q_{Ht}^b \left(B_{Ht}^A - b_{Ht}^A\right) \quad (41)$$

the budget contraint would not be (12') but

$$\begin{aligned} P_t c_t^A + NFA_t^A &= w_t - q_{Ht}^e \left(\chi_{Ht} - \chi_{Ht-1}\right) + d_{Ht}\chi_{Ht-1} + q_{Ht}^b B_{Ht}^A - B_{Ht-1}^A - \left(R_{Ht} - r_{Ft}\right)\omega_{Ht-1}^b \\ &\quad - \left(R_{Ft} - r_{Ft}\right)\omega_{Ft-1}^b + \left(r_{Ht} - r_{Ft}\right)\omega_{t-1}^e + r_{Ft} NFA_{t-1}^A \end{aligned} \quad (42)$$

And the corresponding collateral constraint would not be (13') but

$$\left(1 - \kappa_t\right)\left(q_{Ht}^b b_{Ht}^A + q_{Ft}^b b_{Ft}^A\right) + \kappa_t q_{Ht}^b B_{Ht}^A \leq \kappa_t \left(NFA_t^A + q_{Ht}^e \chi_{Ht}\right) \quad (43)$$

Written this way, the collateral constraint shows the redundancy of a comparison between bonds and equities quite clearly. In fact, given (41) and being $B_{Ht}^A = b_{Ht}^A + b_{Ht}^{*A}$, equation (43) reduces to

$$q_{Ht}^b b_{Ht}^A + q_{Ft}^b b_{Ft}^A \leq \kappa_t \left[q_{Ft}^e k_{Ft} - q_{Ht}^e \left(\chi_{Ht} - k_{Ht}^A\right) + q_{Ht}^e \chi_{Ht}\right]$$

So defining NFE^A and NFB^A as above, I can write the collateral constraint as shown in (13'). In the equilibrium where this constraint is binding, the debt-to-asset ratio is

$$\frac{NFB_t^A + q_{Ht}^b B_{Ht}^A}{NFE_t^A + q_{Ht}^e \chi_{Ht}} = \kappa_t$$

implying that both (39) and (40) amount to

$$E_t \lambda_{t+1}^A \left(R_{Ht+1} - r_{Ft+1}\right) = E_t \lambda_{t+1}^A \left(R_{Ft+1} - r_{Ft+1}\right) = \frac{\mu_t}{\beta \left(c_t^A\right)} \left[\frac{NFB_t^A + q_{Ht}^b B_{Ht}^A}{NFE_t^A + q_{Ht}^e \chi_{Ht}} - 1\right]$$

The interpretation of this result is as follows. There is no choice between a given bond and a given equity claim, so what remains is a choice between equities (equation (37)) and a coice between bonds (equation (36)). The latter is obtained taking the difference between (39) and (40). ■

Remark 2. The total market value of pledgeable assets is only the starting input for computing the home and foreign shares of bond portfolios. In other words, the total amount of collateral available in each country represents the total amount of tradable bonds issued by residents of that country, but it has no implications on the diversification of bond portfolios.

Under binding collateral constraints, the right hand side of equation (13') determines how much home active investors can borrow. In the steady state, borrowing equals $\bar{q}^b \bar{B}^A$, where $\bar{B}^A = \left(\bar{b}_H^A + \bar{b}_F^A\right)$ by symmetry. This confirms that one does not need to determine both an home bond share and a foreign bond share as in (42). For given collateral, the two shares are interdipendent ($\bar{\omega}_F^b = -\bar{\omega}_H^b = -\bar{\omega}^b$), the budget constraint reduces to (12') and one can find the level of diversification for the overall equilibrium portfolios, $\bar{\omega}^e, \bar{\omega}^b$, as detailed in appendix A.2. In particular, $\bar{\chi}$ (per-capita tradable equities) is found as a result of the allocation of capital between backyard sector and firms, occurring in each country. Once $\bar{\chi}$ has been determined, also $\bar{B}^A$ can be found. This means that the allocation of capital between sectors of an economy determines the total amount of tradable collateral but not also its cross-country diversification. This is solved endogenously by $\bar{\omega}^b$. ■

Two are the main implications of these results. The first is that, in the present model collateral constraints split agents' net foreign assets in two components, NFE_t^A and NFB_t^A. One state variable is then replaced by two states, and its behaviour can be recovered easily as $NFA_t^A = NFE_t^A - NFB_t^A$. In addition, active investors' net foreign assets represents country-wide net foreign assets because the passive investors of any given country only purchase the bonds expressed in that country good, regardless of borrowers' residency. Finally, since collateral constraints are at work (symmetrically) in both countries, the usual clearing conditions for international transactions, $NFA_t^A + NFA_t^{*A} = 0$, now needs to be satisfied for both the pledgeable and the collateralized portion of net foreign assets: respectively, $NFE_t^A + NFE_t^{*A} = 0$ and $NFB_t^A + NFB_t^{*A} = 0$.

The second interesting implication is a sort of *corollary* and concerns what happens in a model where just one bond is traded in international markets (Devereux and Yetman, 2010). In this case, investors do not have opportunities to diversify risk in funding, so they borrow as much as their leverage constraints allow. For example, equation (28) implies that home investors' borrowing is $b_t^A = \kappa \left(q_{Ht}^e k_{Ht}^A + q_{Ft}^e k_{Ft}^A \right)$. Then, given home and foreign collateral constraint, the total amount of funds that home and foreign savers must supply is determined by the debt market clearing condition (29): $(1-n)\left(b_t^P + b_t^{*P}\right) = -n\left(b_t^A + b_t^{*A}\right)$. The unique variable that can guarantee this result is clearly the world interest rate. For a given shock to investors' collateral portfolio, κ determines how much the total supply of loans, $b_t^P + b_t^{*P}$, must adjust to satisfy the market clearing condition (29). So, when just one bond is traded, the world rate and κ can display a strong link. In contrast, given equations (30)-(31) and (38), here is not a specific interest rate that matters: investors choose their bond portfolio considering the difference between interest rates, so it is at most this difference that can have an effect on the collateral constraint.

5.2 Equilibrium Portfolios

Table 4. reports the equilibrium portfolios predicted by the numerical solution of the model[22]. Asset holdings are organized by investors as well as by sectors, and each stock is valued at steady state prices and is divided by economy-wide output.

Starting with the allocation of asset holdings across households, the model indicates that passive investors invest in domestic capital and in domestic bonds. These bonds are issued by both home and foreign active investors, but domestic passive investors can neither purchase foreign capital nor foreign bonds. In the steady state, passive investors' capital equals 20 percent of the total available stock (i.e., 0.8/3.98 %), so the remaining 80 percent is marketable across borders. In other terms, patient households invest 1.79-0.80 in local good bonds, and the share of these funds that goes to local borrowers, as opposed to foreign borrowers, depends on active investors' equilibrium portfolios.

Given these portfolios, active investors in each country receive the most of their external funds from local financiers (0.78), while borrowing in the foreign debt market equals 0.21. Given $\bar{\kappa}$, leveraged investors need a bit more than 3 times as much capital as total debt in order to borrow against collateral: in market value terms, 3.18. Once again, these households show a tendency to take bigger positions in the domestic economy than abroad: local firms receive 1.88, while foreign firms 1.3.

The allocation of assets across sectors confirms what just said in more aggregate terms, as passive investors run the backyard sector and active investors own the stock of capital used to produce traded goods. It follows that the latter sector is debtor to the prior. Interestingly, the last four lines of the table show how leverage works in the steady state. The loan granted by backyard producers allow the traded good sector to increase its activity beyond the total net worth of its owners (3.18 of capital against a net worth of 2.19).

[22] I use standard Matlab functions for the analysis of linear systems.

In terms of international diversification, the main message so far is that active investors display local bias in both bond- and equity-holdings. In Table 5, I quantify this home bias and determine what is its effect on the willingness of financiers to grant collateralized loans, depending on the type of pledged equity. The computation is detailed at the bottom of the table. In addition, I get some insights on the hedging properties of bonds and equities by means of some sensitivity analysis.

The key results in Table 5. can be summarized as follows. First, in equilibrium, leveraged investors are chracterized by a 60 percent home equity bias and a 79 percent home funding bias. Second, in spite of the local equity bias, the international diversification attenuates the effect of foreign collateral on counterparty risk. Finally, the bond portfolio captures the ToT (or RER) risk implicit in leveraged borrowing, stabilizing the equity portfolio even along this dimension. The rest of this section is devoted to the interpretation of these three findings.

First, the home bias result is in line with previous literature. As in Coeurdacier et al. (2010), the home equity bias is justified by the home bias in the investment expenditures to accumulate capital over time. This means that capital accumulation breaks the perfect correlation between capital income and non-capital income, regardless of the presence of binding collateral constraints[23].

Second, the home equity bias allows to interpret the SMM estimation of (34) (and its foreign counterpart) and to assess the effect of diversification on κ_t (κ_t^*) as proxy for borrowers' counterparty risk. By construction, borrowers pledge an entire portfolio of assets, which is made of both home and foreign equities. Thus, conditional on a given portfolio allocation, the SMM estimation of (34) suggests how much financiers revise their willingness to lend to home active investors in case of a change in the value of either home equity (ψ), foreign equity (ψ^*) or both. According to the estimation results, $\psi < \psi^*$, meaning that the debt-to-asset ratio of each borrower is more sensitive to the riskiness of foreign collateral than to that of local collateral. For example, the estimation suggests that a loss in the market value of foreign collateral cuts home agents' borrowing by 0.185, while an equivalent loss on home collateral reduces it by 0.126.

This result is clearly driven by cross-border risk diversification. Assume, for simplicity, that the function $g = [\cdot]$ defining ψ, ψ^* as a given transformation of portfolio shares is linear. On the basis of a first-order solution, this reveals that

$$\begin{aligned}
\psi &= g\left[\frac{q^e_{Ht}k^A_{Ht}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}}\right] = 0.213 \times \frac{q^e_{Ht}k^A_{Ht}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}} = 0.126 \\
\psi^* &\equiv g\left[\frac{q^e_{Ft}k^A_{Ft}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}}\right] = 0.450 \times \frac{q^e_{Ft}k^A_{Ft}}{q^e_{Ht}k^A_{Ht} + q^e_{Ft}k^A_{Ft}} = 0.185
\end{aligned}$$

Intuitively, in absence of the level of risk diversification achieved in equilibrium, the riskiness of foreign

[23] When capital income is perfectly correlated with non-capital income, the prediction of international portfolio models goes back to Baxter and Jermann (1997): investors tend to use foreign equities as a protection against their labour income risk.

collateral would affect κ_t more than twice as much as the riskiness of local collateral. Yet, note that this does not imply that active investors' portfolio choice accounts for these differences in the riskiness of home and foreign collateral. Borrowers do not have any incentive to account for the riskiness specific to each asset: given the collateral constraint (equation (13')), financiers lend against a synthetic measure of borrowers' counterparty risk κ_t because they receive an overall asset portfolio, with international diversification attenuating asset-specific effects (equation (34))[24]. For an alternative solution, see Trani (2012).

Finally, the home funding bias (and its sensitivity to model parameters) reflects how active investors cope with the ToT risk affecting their balance sheets. In equilibrium, active investors borrow as much as 79 percent of their total debt from local financiers because, among other reasons, bonds are the unique assets that they use to hedge the ToT risk, although this drives a wedge even between home and foreign equity prices. One reason for this is that debt instruments are specifically used to satisfy the no-arbitrage condition between home and foreign loan rates (equation (36)). Another reason is the fact that, under binding constraints, only the total size of the pledged collateral (not also its diversification) affects the choice between home and foreign funding. Consequently, bond holdings are particularly and univocally sensitive to the strength of the ToT risk, as implied by model parameters.

The parameters governing the share of local goods versus foreign goods in consumption and investment have a positive effect on the bias in local funding. That is, the tendency to borrow more heavily at home than abroad is larger, the larger γ, γ_I (the home bias in consumption and investment) and θ, θ_I (the elasticity of substitution between home and foreign goods in consumption and investment). In contrast, the results are more mixed for the home equity bias, which is moreover far less responsive to changes in model parameters than the bond shares. Coeurdacier and Gourinchas (2011) conduct a careful study on similar results, and I remand to the separate technical appendix for the specific numbers obtained in the case of changes to $\gamma, \theta, \gamma_I, \theta_I$. The new result here concerns how the level of leverage sought by active investors affects the endogenous diversification of their portfolios. To start with, I increase the steady state debt-to-asset ratio by 32 percent to 0.41, keeping $\psi/\bar{\kappa}$ fixed (case a of the test "Greater Leverage" in Table 5.). This way equilibrium leverage increases from 1.45 to 1.69, but the impact of home and foreign collateral on counterparty risk remains unchanged. I find an important confirmation for the theoretical prediction obtained in (30)-(31). According to this prediction, there should be a link between κ_t and both of the borrowing rates, so this link is expected to affect active investors' willingness to diversify their debts across countries. Quantitatively, only the equilibrium bond share reacts (and markedly) to the increase in leverage, while the equity share remains unaffected. The remaining two exercises are meant to check whether these results are not simply due to the fact that I have initially kept the weight of ψ on $\bar{\kappa}$ constant. As the table shows (cases b,c of the test "Greater

[24] The consequence is that the unique objective behind the satisfaction of the portfolio Euler for equities (equation (39)) is to eliminate any excess return differential across borders, as already found by Devereux and Yetman (2010).

Leverage"), the liability portfolio continues to have the same hedging properties as before.

To sum up, the main result of this section is that the ToT risk interacts with borrowers' leverage causing local bias in funding. And in turn this position in bonds eliminates the most of the effect of changes in the RER on other assents. A clear caveat is that the predictions of the model are likely to overestimate the sensitivity of bonds to model parameters. The reason for this is that the model does not involve other bonds than those used by active investors to attract funds under borrowing limits. However, introducing unconstrained bonds would not affect the results qualitatively.

6 International Transmission and Predicted NFAs

From now on, I study the dynamic properties of the model described in section 4. The goal of this analysis is not to provide a complete representation of deleveraging and of the ensuing international transmission mechanism. Since I build on Devereux and Yetman (2010), the major insights of their analysis are valid here as well. I focus instead on the potential contribution of the current framework to their findings on how binding borrowing limits transmit idiosyncratic shocks to foreign countries. The two new features here are: the international integration between different markets for debt securities and the effects of shocks on counterparty risk. For clarification purposes, I shall at times compare the predictions of the present model with those of a framework such as Devereux and Yetman's.

I solve their model, focusing on the version in which there is a unique worldwide bond market (equation (29)). Devereux and Yetman's model does not involve home and foreign bonds as well as the factors behind home bias (i.e., capital accumulation and home bias in consumption). So although their model do involve home and foreign equities, it does not reproduce home equity bias endogenously, which is simply not their objective. Since home equity bias is instead an endogenous feature of my model, I introduce in their framework a second order transaction cost τ, which affects the portfolio choice in the way described by Tille and Van Wincoop (2010). I calibrate τ so that the equilibrium home equity bias in the framework à la Devereux and Yetman is the same as the one found here (roughly 60 percent).

Consider then a negative shock to home country productivity (graph not included in the paper[25] but in the separate technical appendix). The productivity shock causes a fall in the price of home equities. In turn, this price effect sets in motion the international transmission mechanism provided by the financial accelerator in open economy. The mechanism works through binding borrowing constraints under internationally diversified assets. Home equities are indeed present in both home and foreign international investors' portfolios, so the decrease in the value of those equities translates into a wealth loss. As a consequence, borrowing constraints in (13) tighten, and the initial shock to home equities is transmitted to foreign equities. This way, the wealth loss amounts to a decrease in the value of collateral

[25] See also Figure 4., which shows a similar picture as the one in the saparate appendix, with the only difference that the former graph is obtained shutting down the reaction of debt-to-asset ratios to equity prices.

portfolios, so the supply of credit decreases and the initial shock propagates worldwide, in the form of a generalized deleveraging in all long positions.

The novelty here is twofold, as this deleveraging cycle entails: 1) a real appreciation of home goods versus foreign goods; 2) a fall in borrowers' debt-to-value ratios. Both effects have consequences on asset prices, including the market value of both bonds and equities worldwide. Due to these new price effects, the net foreign assets of the home country (foreign country) must decrease (increase) on impact and increase (decrease) afterwards, as model variables go back to equilibrium. A one-bond model with constant debt-to-asset ratios such as Devereux and Yetman's would predict exactly the opposite. Figure 2. shows what just said, taking into account only the home country (where the shock originates).

In Devereux and Yetman, the fall in home country productivity causes a quite marked trade balance (TB) deficit (third panel) but *first order* valuation and income balance effects (VE) of an opposite sign (fourth panel). This means that, given equilibrium portfolios, foreign investors suffer a wealth loss on their holdings of home equities which is relatively bigger than the devaluation of home agents' holdings of foreign equities. Through the mechanism I shall describe in the rest of the paper, the model here reverts this conclusion because the reaction of the ToT $1/p_{Ft}$ negatively affects cross-border funding and the immediate increase in counterparty risk (mimicked by a fall in κ_t) have perverse consequences for equity prices. Hence, the net foreign assets of the home country can decrease (first panel), even if the fall in TB is quite contained (third panel).

Interestingly, the predictions of the present model match quite well the behaviour of the U.S. (Net) International Investment Position (IIP) during the 2007-09 crisis, which are reported in Figure 3. The vertical (and dashed) line in the graph denotes the year in which the crisis started: 2007. According to the data, the beginning of the crisis brings about a negative valuation effect for the U.S economy, as the fall in its IIP is not entirely matched by its current account (and TB). Actually, the TB would adjust upward in the following year. The model here cannot capture this upward adjustment, and the sequencing of effects is not exactly the same as in the data. However, the TB deficit it predicts is much smaller than in Devereux and Yetman, and the *first order* VE it generates revert those yielded by the latter model.

6.1 Integrated Markets for Debt Secuties

In this section, I focus only on the contibution of multiple bonds to the international transmission mechanism. Accordingly, I shut down the reaction of κ_t, κ_t^* to the riskiness of collateral assets, conditional on the equilibrium portfolios - i.e., equation (34) and its foreign counterpart. In this case, a negative productivity shock in the home country yields the responses shown in Figure 4.

According to this graph, the productivity shock at home has particularly severe effects on all equity prices, which comove almost perfectly across borders (first panel). So home and foreign active investors' collateral must fall, but home equity bias implies that the wealth loss is bigger for domestic agents (second panel). This leads to a margin call, as their capability to borrow falls in line with the decrease in collateral (third panel), and the supply of loans adjust accordingly (fifth panel). A deleveraging cycle starts, and panels 7, 9 and 10 show its macroeconomic effects: especially significant is the decrease in the demand for the investment goods produced by firms in the home country (and, by symmetry, for all investment spending at home).

Panels 4, 5, 8 (of Figure 4) contain the main results on how the integration between home and foreign bond markets affect the transmission of shocks. The negative productivity shock causes a real appreciation of home country goods, which drives a wedge between home and foreign loan rates. This wedge is apparently at odds with the *shadow* loan premiums paid by home and foreign borrowers (which are driven by the respective shadow prices of collateralized borrowing, μ_t and μ_t^*), because both increase symmetrically across borders[26]. I focus on these price effects more specifically in Figure 5.

This graph suggests that the increase in the home ToT has two effects. First, the ToT heightens the transmission between home and foreign variables. The reaction of the ToT magnifies the effect of the shock on equity prices, with a tendency to increase their cross-country correlation with respect to what happens in Devereux and Yetman (labeled as "DY Model"). Second, and more importantly, the ToT raises the difference between the costs of home funding versus the cost foreign funding and disconnects the behaviour of interest rates from the value assumed by equilibrium debt-to-asset ratios. The third panel in Figure 5. confirms the corollary at the end of section 5.1: in "DY Model", the dynamics of the interest rate depend on $\bar{\kappa}$, even if their leverage constraint limits the total size of debt that investors can obtain as opposed to the repayment costs (equation (28)). When $\bar{\kappa}$ is calibrated as in my model (0.31), the effect of the negative shock to home country productivity on the world interest rate is even sligthly positive. Only with higher debt-to-asset ratios (e.g., $\bar{\kappa} = 0.5$, which is one of Devereux and Yetman's assumptions), the world rate on loans can decrease. On the other hand, allowing for an endogenous choice between home and foreign liabilities, the model here eliminates this link between interest rate and leverage: dynamically, the mechanism works through risk premiums in the way I pass to explain.

The contrast between loan premiums and interest rate differentials shown in Figure 4 (fourth and fifth panels) is due to the fact that, under financial integration, the shadow price of collateralized borrowing is just one component of the *overall* premiums paid on loans. Home and foreign interest rates must reflect home and foreign financiers' stochastic discount factors (equations (20)-(22))[27]. The overall premium on funding is then obtained using these discount factors together with active investors'

[26] See section 3.3.2

[27] Formally:

$$E_t R_{Ht+1} = \frac{1}{E_t \Lambda^P_{t,t+1}} \quad ; \quad E_t R_{Ft+1} = \frac{1}{E_t \Lambda^{*P}_{t,t+1}} - \frac{cov_t\left(\Lambda^{*P}_{t,t+1}, p_{Ft+1}\right)/q^b_{Ft}}{E_t \Lambda^{*P}_{t,t+1}}$$

consumption Eulers. For home agents (equations (14)-(15)), I get

$$GP^{A}\left(b_{Ht}\right) = \underbrace{\frac{\mu_t/\lambda_t^A}{E_t\Lambda_{t,t+1}^A}}_{\text{shadow premium}} \tag{44}$$

$$GP^{A}\left(b_{Ft}\right) = \underbrace{\frac{\mu_t/\lambda_t^A}{E_t\Lambda_{t,t+1}^A}}_{\text{shadow premium}} + \underbrace{\frac{cov_t\left(\Lambda_{t,t+1}^A, p_{Ft+1}\right)/q_{Ft}^b}{E_t\Lambda_{t,t+1}^A} - \frac{cov_t\left(\Lambda_{t,t+1}^{*P}, p_{Ft+1}\right)/q_{Ft}^b}{E_t\Lambda_{t,t+1}^{*P}}}_{\text{ToT component}} \tag{45}$$

In (44)-(45), I define the loan premium as *guarantee premium* (GP) because it is the premium imposed by borrowing under collateral (which can be considered as a guarantee or a loan covenant). It is clear that, for active investors, only the premium on domestic debt is uniquely determined by the shadow cost of borrowing μ_t (equation (44)). The cost of foreign debt involves a ToT component. This component is represented by how the ToT correlates with borrowers' pricing kernel relative to how it correlates with lenders' pricing kernel.

After the fall in home productivity, $1/p_{Ft}$ icreases, and so do $\Lambda_{t,t+1}^A, \Lambda_{t,t+1}^{*P}$. Therefore, both covariances in (45) are expected to be negative, meaning that $GP^A\left(b_{Ft}\right)$ is expected to fall with the effect of the ToT on borrowers' discount factors (first term of the ToT componet) but to increase with the effect of the ToT on lenders' discount factor (second term of the ToT componet). But since passive investors are more patient than active investors, the second effect prevails: $GP^A\left(b_{Ft}\right)$ is expected to increase by more than $GP^A\left(b_{Ht}\right)$.

The effect of the negative productivity shock on loan premiums imply that foreign financiers are the first lenders that refuse to grant loans to home country leveraged investors - and conversely for foreign borrowers. To show this, I compute "differences" between the predictions of the model here and those of the single-bond model of Devereux and Yetman and report the result in Figure 6. If the difference falls in the negative region (i.e., below the zero line), the variable plotted reacts by more in the model here than in Devereux and Yetman. The opposite is true when the difference falls in the positive region (i.e. above the zero line). One can see that, on the first dates, the supply of funds by home financiers is quite similar across models, while foreign financiers curb their loans by more in the model here than in Devereux and Yetman (third panel). And from the second panel, foreign financiers reduce not only their credit to local active investors, but also their credit to borrowers in the home country. That is, cross-border funding experiences a backstop after a negative macroeconomic shock to one economy (which is confirmed by how active investors adjust their consumption across borders, in the last panel).

In conclusion, the major effect of the integration between markets for debt securities is that the ToT affects the internationally diversified debt portfolio of leveraged investors. A new effect on the international transmission of shocks follows. Given equations (30)-(31) and the equilibrium loan portfolio, the gap between bond prices created by the increase in the home ToT (fifth panel in Figure 5.) implies that the margin call received by active investors is different from that received by foreign investors. And since also equity prices comply with the movements in the ToT, the collateral portfolios of home and foreign agents react in line with the different margin calls they receive. This justifies the impulse response in the first panel of Figure 6: the devaluation of foreign borrowers' collateral is less pronounced here than in a one-bond model, while the opposite is true for home agents' collateral. In net terms, NFB_t^A must decrease because foreign funding react more than domestic funding, and NFE_t^A moves in the opposite direction than predicted by Devereux and Yetman's NFA_t. Since NFE_t^A falls by more that NFB_t^A, NFA_t^A must decrease as shown in Figure 7. (first panel).

7 The Behavior of Debt-to-Asset Ratios

7.1 The Reaction to Macroeconomic Shocks

The analysis so far highlights the ToT effect on interest rate differentials and equity prices. However, the sole ToT effect attributes a somehow passive role to equity prices. For equilibrium purposes, the value of home and foreign collateral portfolios must adjust to the changes in cross-border funding. On the other hand, here I show that wholesale funding can imply a more independent role for equity prices through the reaction of financiers to changes in counterparty risk.

According to equations (16)-(17), the effect of binding collateral constraints is that borrowers discount future capital gains and dividends more heavily than under non-binding constraints:

$$q_{it}^e = E_t \underline{\Lambda}_{t,t+1}^A \left(q_{it+1}^e + d_{it+1} \right) \ \forall i = H, F$$

where the "distorted" pricing kernel is

$$\underline{\Lambda}_{t,t+1}^A \equiv \frac{\beta \left(c_t^A \right) E_t \lambda_{t+1}^A}{\lambda_t^A - \mu_t \kappa_t} = \frac{\Lambda_{t,t+1}^A}{\left(1 - \frac{\mu_t}{\lambda_t^A} \kappa_t \right)} > \Lambda_{t,t+1}^A \tag{46}$$

As a consequence, equilibrium equity prices are distorted downward (Aiyagari and Gertler, 1999; Mendoza, 2010):

$$\begin{aligned} q_{Ht}^e &= E_t \left[n \underline{\Lambda}_{t,t+1}^A + (1-n) \Lambda_{t,t+1}^P \right] q_{Ht+1}^e + n E_t \underline{\Lambda}_{t,t+1}^A d_{Ht+1} + (1-n) E_t \Lambda_{t,t+1}^P \nu z \left(k_{Ht}^P \right)^{\nu - 1} && (47) \\ q_{Ft}^e &= E_t \left[n \underline{\Lambda}_{t,t+1}^A + (1-n) \Lambda_{t,t+1}^{*P} \right] q_{Ft+1}^e + n E_t \underline{\Lambda}_{t,t+1}^A d_{Ft+1} + (1-n) E_t \Lambda_{t,t+1}^{*P} \nu z \left(k_{Ft}^{*P} \right)^{\nu - 1} && (48) \end{aligned}$$

where I have used passive investors' pricing equations as well.

The interpretation is as follows. Active investors' pricing kernel (46) is affected by both the shadow price of borrowing (in terms of consumption), μ_t / λ_t^A, and the contractual limit imposed on this debt, κ_t. When agents' debt-to-asset ratio is a constant κ, only the shadow price of borrowing matters. As a consequence, a negative productivity shock prompts active investors to discount the future even more heavily than before the shock. The reason for this is that the shock has perverse effects on the value of collateral through d_{Ht} in (47), so lenders curb credit and μ_t/λ_t^A must increase. This is the *loss spiral* effect of the shock (Brunnermeier, 2009). But when debt-to-asset ratios are time-varying as in (34), the effects of the shock are more complex.

In fact, lenders react to the fall in equity prices, believing that contracting with a given active investor is riskier as her collateral is less valueable. As a consequence, κ_t decreases partly contrasting the reaction of μ_t/λ_t^A. The combined effect on $\underline{\Lambda}_{t,t+1}^A$ is ambiguous and depends on which variable reacts more forcefully and is more influential between κ_t and μ_t/λ_t^A. In other words, the fall in the debt-to-asset ratios tighten the haircuts so that a *margin spiral* accompanies the loss spiral deleveraging (Brunnermeier, 2009).

Let us now see how these theoretical intuitions show up in the dynamic behaviour of the model, looking at how the reaction of μ_t/λ_t^A and κ_t combine together and, thus, how debt-to-asset ratios affect equity prices. But note that the interaction between μ_t/λ_t^A and κ_t is even more complex, as the former depends on κ_t through the collateral constraints. The tighter the constraint, the greater the shadow price of borrowing. So to start with, in Figure 7. I plot the reaction of NFE_t^A and NFB_t^A to the productivity shock assuming that κ_t, κ_t^* can react to home and foreign equity prices (and comparing this case with that of a constant κ).

Figure 7. shows that, after a shock to home productivity, NFA_t^A falls largely because of an endogenous reaction of equity prices to κ_t, κ_t^*[28]. The model with constant debt-to-asset ratios predicts that NFA_t^A must decrease because of the ToT effect (first panel). However, this fall is not quantitatively large because the ToT mainly affects bond prices. In contrast, the reaction of financiers has marked effects on equity prices and small effects on bond prices. Subtracting the reaction of variables under constant debt-to-asset ratios from their reaction under time-varying ratios, I find that the differential effect of the variation in κ_t, κ_t^* on NFE_t^A is much stronger than its differential effect on NFB_t^A (third panel). Now, NFA_t^A decreases around 1.5 times more than before (second versus first panel).

To further interpret this result, consider that Van Wincoop (2011) doubts that, in absence of factors such as risk shocks, binding leverage constraints can *per se* cause similar effects as those observed in 2007-09. Here, quantitatively important is the impact reaction of κ_t, κ_t^*, interpreted as counterparty risk.

The third panel of Figure 7. suggests what is the complete mechanism at work, that is, how κ_t and μ_t/λ_t^A - combined - affect the pricing kernel in (48). The fact that NFE_t^A falls on impact obviously means that the fall in κ_t is very strong, but its fall is more than compensated by the increase in μ_t/λ_t^A and the pricing kernel must increase. However, on later dates the effect works in the opposite direction: the pricing kernel contracts, quickening the adjustment of NFE_t^A back to equilibrium.

[28] See also the first panel in Figure 2.

This reversion of sign is confirmed by the ex post behaviour of the other model variables, apart from those directly connected with the collateral portfolios - especially influenced by borrowing limits effects (equations (13)) or market clearing effects (equations (26)). Figure 8. reports the results of an experiment, where I analyze how impact and cumulative multipliers of various variables behave as ψ is progressively incresead from 0 to its estimated value. Cumulative multipliers are computed over the first five dates after the shock.

The main message from this graph is as follows. After revising borrowers' counterparty risk upward, financiers curb credit very sharply and over more dates (first panel). So, the greater the adjustment of κ_t, κ_t^* to equity prices, the smaller the amount of funds that active investors can borrow (fifth panel). But this heightened credit crunch has a sort of "market discipline effect": while on impact macroeconomic variables (e.g., investment) and the shadow loan premium increase, they benefit from the adjustment in κ_t over the longer run. The general consequence is that recessions (and their worldwide impact) are deeper but less long-lasting if debt-to-asset ratios are time-varying than if they are not.

7.2 Dynamics with Additional Shocks

The conclusion of the previous section - that the reaction of financiers to economic shocks eventually dampens their effects and their transmission across borders - is not applicable to all shocks. In general, the dampening effect produced by changes in κ_t, κ_t^* on later stages after a shock is true for macroeconomic innovations but not for financial innovations. Following Jermann and Quadrini (2012), the financial shocks affecting the model under analysis are the exogenous innovations to κ_t, κ_t^*; that is, $\epsilon_{\kappa t}, \epsilon_{\kappa^* t}$.

In Figure 9., I present simulation results[29] for various combinations of shocks. Clearly, the graph shows (for some of the most relevant variables) that the simulated time-series are more volatile when macroeconomic shocks are accompanied by (some exogenous) uncertainty about counterparty risk. And this is not the mere result of adding shocks together.

The baseline case is represented by the behaviour of macroeconomic variables under productivity shocks, which have been considered so far (cyan, dashed line; long dashes). Adding investment shocks (black, dashed line; short dashes) does not have any effect on macroeconomic and financial variables, except for their contained effect on investment expenditures themselves. In contrast, adding the shocks to debt-to-asset ratios (red, continuous line) increases the volatility of all macroeconomic variables, and this increase is even more marked if one considers the initial level of leverage (red, dash-dotted line). Initial leverage is taken into account scaling the financial shocks by $\bar{m}/\bar{\kappa}$ (equation (34)).

These results are somewhat in line with recent financial literature, which thus suggests a possible

[29] I conduct a Monte Carlo experiment, setting it up as follows. I simulate time-series of 120 datapoints. I pass them through the Hodrick-Prescott filter. Finally, I drop the first 50 observations. To construct the variables ultimately used I repeat this process for 600 times.

interpretation. In Geanakoplos (2009), the endogenous changes in leverage have stabilizing properties for the most of the possible economic shocks but not for shocks to the tail distribution of asset returns. From another viewpoint, Brunnermeier and Pedersen (2009) - from whom I partly draw - find that margins are prevalently stabilizing when financiers have enough information about the fundamentals of the economy but not when they are uninformed. Therefore, the reaction of debt-to-asset ratios at work here can stabilize economic variables *over time* if the world economy is hit by shocks that do not concern the risk of lending to a certain borrower. In this case, κ_t, κ_t^* follow the mechanism implicit in RE equity pricing (solving forward (16)-(17), (21), (23)). But if macroeconomic shocks are accompanied by doubts about borrowers' true creditworthiness and/or about the riskiness of their collateral, then the behaviour of κ_t, κ_t^* can "destabilize" the economy. The uncertainty faced by financiers is passed onto equity prices through active investors' "distorted" discount factor (46) - affecting the forward solution. Clearly, this uncertainty propagates to the whole economy because of the key role of equities as collateral for borrowing and, thus, for financing investment and production across borders.

In terms of international transmission, the interaction between ToT and leverage can be certainly expected to reflect the destabilizing properties of haircuts in case of financial shocks, suggesting once again that risk may have played a role in the propagation the 2007-09 crisis (Van Wincoop, 2011). But the analysis herewith is far from conclusive, and further research is needed.

8 Conclusions

I develop a two-country portfolio model to study the effect of wholesale funding on the international transmission of shocks under collateral constraints. I model two main aspects of these collateralized loan contracts: the fact that the collateralized portfolio of liabilities is diversified across home and foreign securities, and the fact that wholesale financiers react to changes in counterparty risk.

Besides, my framework proposes a way to model and parametrize the debt-to-asset ratio when borrowers do not face limits on single assets but on their internationally diversified portfolios. I also show how the computation of these portfolios simplifies through the effect of financial integration on debt markets, which are characterized by collateralized borrowing.

The presence of multiple bonds to use as liability instruments shows that investors' leverage interacts with the terms of trade (or real exchange rate) risk. This interaction is absent in a model with just one debt market. Such a model links the dynamics of the loan rate to the equilibrium level of debt-to-asset ratios because agents cannot take advantage of the hedging properties of bonds, and borrowing is uniquely dictated by the collateral constraints.

Coherently, the international diversification of funding shows how strong is the cross-border transmission when a perverse shock reduces the extent to which agents can take advantage of these hedging properties of bonds. The reason for this is the ToT effect. The ToT affects equity prices as well, but in this case the role of funding works through the time-variation in debt-to-asset ratios. On impact, this adjustment affects equity prices in a negative way. And this effect - combined with the integration in debt markets - helps the model predict a fall in the net foreign assets of the economy where the shock originates. This matches the U.S. experience in the 2007-2009 crisis. The rest of the adjustment in haircuts can have stabilizing or destabilizing effect, but further research is needed in this case.

References

[1] Adrian, T. and H. S. Shin (2008) "Procyclical Leverage and Value-at-Risk", *Federal Reserve of New York*, Staff Report no.338.

[2] Adrian, T. and H. S. Shin (2009) "Financial Intermediaries and Monetary Economics", *Federal Reserve of New York*, Staff Report no.398.

[3] Aiyagari, S. R. and M. Gertler (1999) "Overreaction of Asset Prices in General Equilibrium", *Review of Economic Dynamics*, 2, 3-35.

[4] Ayar, S. (2011) "How Did the Crisis in International Funding Markets Affect Bank Lending? Balance sheet evidence from the United Kingdom", *Bank of England,* Working Paper No. 424.

[5] Baxter, M. and U. J. Jermann (1997) "The International Diversification Puzzle is Worse Than You Think", *American Economic Review*, 87(1), 170-80.

[6] BIS (2008) "Guidelines to international locational banking statistics", Monetary and Economic Department, *Bank for International Settlments.*

[7] Brunnermeier, M. (2009) "Deciphering the Liquidity and Credit Crunch 2007-2008", *Journal of Economic Perspectives*, 23(1), 77-100.

[8] Brunnermeier, M. and L. H. Pedersen (2009) "Market Liquidity and Funding Liquidity", *Review of Financial Studies*, 22(6), 2201-2238.

[9] Canova, F. (2007) *Methods for Applied Macroeconomic Research*, Princeton University Press.

[10] Carlstrom, C. T. and T. S. Fuerst (1997) "Agency Costs, Net Worth and Business Fluctuations: a Computable General Equilibrium Analysis", *American Economic Review*, 87(5), 893-910.

[11] Cetorelli, N. and L. S. Goldberg (2010) "Global Banks and International Shock Transmission: Evidence from the Crisis", *NBER Working Paper*, 15974.

[12] Coeurdacier, N. and P.-O. Gourinchas (2011) "When Bonds Matter: Home Bias in Goods and Assets", *NBER Working Paper*, 17560.

[13] Coeurdacier, N., R. Kollmann and P. Martin (2007) "International Portfolios with Supply, Demand and Redistributive Shocks", in Clarida and Giavazzi (2009) *NBER International Seminar on Macroeconomics 2007*, NBER Books.

[14] Coeurdacier, N., R. Kollmann and P. Martin (2010) "International Portfolios, Capital Accumulation and Foreign Asset Dynamics", *Journal of International Economics*, 80(1), 100-112.

[15] Corsetti, G., L. Dedola and S. Leduc (2008) "International Risk Sharing and the Transmission of Productivity Socks", *Review of Economic Studies*, 75(2).

[16] Dedola, L. and G. Lombardo (2010) "Financial Frictions, Financial Integration and the International Propagation of Shocks", NBER Summer Institute 2010, IFM, *mimeo*.

[17] Devereux, M. B. and A. Sutherland (2007) "Solving for Country Portfolios in Open Economy Macro Models", *HKIMR Working Paper* 16.

[18] Devereux, M. B. and A. Sutherland (2011) "Country Portfolios in Open Economy Macro Models", *Journal of European Economic Association*, 9(2), 337-369.

[19] Devereux, M. B. and J. Yetman (2010) "Leverage Constraints and the International Transmission of Shocks", *Journal of Money, Credit and Banking*, 42(6).

[20] Duffie, D. and K. J. Singleton (1993) "Simulated Moments Estimation of Markov Models of Asset Prices", *Econometrica* 61(4), 929-952.

[21] Fisher, J. D. M. (2006) "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks", *Journal of Political Economy*, 114, 413-452.

[22] Geanakoplos, J. (2009) "The Leverage Cycle", in D. Acemoglu, K. Rogoff and M. Woodford, eds., *NBER Macroeconomic Annual 2009*, vol. 24, 1-65, University of Chicago Press.

[23] Goldberg, L. S. (2009) "Understanding Banking Sector Globalization", *IMF Staff Papers*, 56(1), 171-197.

[24] Gorton, G. B. (2009) "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007", *Federal Reserve Bank of Atlanta*, Conference Paper.

[25] Gorton, G. B. and G. G. Pennacchi (1990) "Financial Intermediaries and Liquidity Creation", *The Journal of Finance*, 45(1), 49-71.

[26] Gorton, G. B. and G. G. Pennacchi (1995) "Banks and Loan Sales. Marketing Non-Marketable Assets", *Journal of Monetary Economics*, 35(3), 389-411.

[27] Gorton, G. B. and A. Metrick (2009a) "Haircuts", *NBER Working Paper* 15273.

[28] Gorton, G. B. and A. Metrick (2009b) "Securitized Banking and the Run on Repo", *NBER Working Paper* 15223.

[29] Gourinchas, P.-O., H. Rey and K. Truempler (2011) "The Financial Crisis and the Geography of Wealth Transfers", *NBER Working Paper* 17353.

[30] Greenwood, J., Z. Hercowitz and P. Krusell (1997) "Long Run Implications of Investment-Specific Technology Shocks", *American Economic Review*, 87(3), 342-362.

[31] Hart, O. D. and J. Moore (1995) "Debt ad Seniority: an Analysis of the Role of Hard Claims in Constraining Management", *American Economic Review*, 87(3), 567-585.

[32] Iacoviello, M. (2005) "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle", *American Economic Review*, 95(3), 739-764.

[33] Iacoviello, M. and R. Minetti (2006) "International Business Cycle with Domestic and Foreign Lenders", *Journal of Monetary Economics*, 53(8), 2267-2282.

[34] Jermann, U. J. and V. Quadrini (2012) "Macroeconomic Effects of Financial Shocks", *American Economics Review*, 102(1), 238-71.

[35] Judd, K. L., F. Kubler and K. Schmedders (2002) "A Solution Method for Incomplete Markets with Heterogeneous Agents", Stanford University, *mimeo.*

[36] Justiniano, A., G. E. Primiceri and A. Tambalotti (2009) "Investment Shocks and the Relative Price of Investment", *Federal Reserve Bank of New York,* Staff Report no. 411.

[37] Kiyotaki, N and J. Moore (1997) "Credit Cycles", *The Journal of Political Economy*, 105(2), 211-248.

[38] Klein, P. (2000) "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model", *Journal of Economic Dynamics and Control*, 24(10), 1405-1423.

[39] Kocherlakota, N. (2000) "Creating Business Cycle through Credit Constraints", *Federal Reserve Bank of Minneapolis Quarterly Review*, 24(3), 2-10.

[40] Kollmann, R., Z. Enders and G. J. Muller (2011) "Global Banking and the International Business Cycles", *European Economic Review*, 55(3), 407-426.

[41] Krugman, P. R. (2008) "The International Financial Multiplier", Princeton University, *mimeo*.

[42] Michaelides, A. and S. Ng (2000) "Estimating the Rational Expectation Model of Speculative Storage: a Monte Carlo Comparison of Three Simulation Estimators", *Journal of Econometrics*, 96(2), 231-266.

[43] McCallum, B. (1998) "Solution to Linear Rational Expectations Models: a Compact Exposition", *Economic Letters*, 61(2), 143-147.

[44] McGuire, P. M. and G. Von Peter (2009) "The US Dollar Shortage in Global Banking", *BIS Quarterly Review*, March, 47-63.

[45] Mendoza, E. (2008) "Sudden Stops, Financial Crises and Leverage: a Fisherian Deflation of Tobin's Q", *Board of Governors of the Federal Reserve System*, International Finance Discussion Paper 960.

[46] Mendoza, E. G. (2010) "Sudden Stops, Financial Crises and Leverage", *American Economic Review*, 100(5), 1941-66.

[47] Mendoza, E. G. and K. A. Smith (2006) "Quantitative Implications of a Debt-Deflation Theory of Sudden Stops and Asset Prices", *Journal of International Economics*, 70(1), 82-114.

[48] Pavlova, A. and R. Rigobon (2008) "The Role of Portfolio Constraints in the International Propagation of Shocks", *Review of Economic Studies*, 75(4), 1215-1256.

[49] Ruge-Murcia, F. (2007) "Methods to Estimate Dynamic Stochastic General Equilirbium Models", *Journal of Economic Dynamics and Control*, 31, 2599-2636.

[50] Sercu, P. and R. Vanpée (2007) "Home Bias in International Equity Portfolios: a Review", *Catholic University of Leuven working paper*, AFI 0710.

[51] Shin, H. S. (2011) "Global Banking Glut and Loan Risk Premium", *Mundell-Fleming Lecture*, 12th Jacques Polak Annual Research Conference, IMF.

[52] Tille, C. and E. Van Wincoop (2010) "International Capital Flows", *Journal of International Economics*, 80(2), 157-175.

[53] Trani, T. (2012) "Country Portfolios with Heterogeneous Pledgeability", The Graduate Institute | Geneva, *mimeo*.

[54] Tsay, R. (2005) *Analysis of Financial Time Series*, John Wiley and Sons.

[55] Van Wincoop, E. (2011) "International Contagion through Leveraged Financial Institutions", *NBER Working Paper* 17686.

Graphs and Tables

Table 1: Parameters Not Governing Borrowers' Margins

Parameter	Description	Value based on ...
		... previous studies
n	number of constrained investors	0.5
ϕ	discount factor parameter	0.022
γ	share of home goods in consumption	0.72
θ	elasticity of substitution in consumption	0.85
γ_I	share of home goods in investment	0.75
θ_I	elasticity of substitution in investment	0.9
σ	CRRA	2
α	capital share in income	0.4
z	fixed productivity in backyard production	1
ν	degree of homogeneity in backyard sector	0.1
δ	rate of capital depreciation	0.1
		... UK and US data
$\overline{R}$	gross rate of interest	1.0418
$\bar{\mu}/\left[\beta\left(\bar{c}^A\right)\bar{\lambda}^A\right]$	guarantee premium	0.001134
		... OECD data
ρ_A	TFP shock: persistence	0.81
ρ_Ξ	investment shock: persistence	0.85
σ_A	TFP shock: volatility	0.015
σ_Ξ	investment shock: volatility	0.017
$\rho\left(\varepsilon_{At}, \varepsilon_{A^*t}\right)$	TFP shocks: cross-country correlation	0.45
$\rho\left(\varepsilon_{\Xi t}, \varepsilon_{\Xi^* t}\right)$	investment shocks: cross-country correlation	0.26

Table 2: Calibrating $\bar{\kappa}$

Chosen Strategy: OECD Data on Lenders		
Passive investors	Share of Equities in Tot. Assets $\frac{\bar{q}^e \bar{k}^P}{\bar{q}^e \bar{k}^P - \bar{q}^b \bar{B}^P}$	Corresponding Debt-to-Asset Ratio
Investment funds	0.49	-
Insurance corporations and pension funds	0.42	-
Model	0.45	0.31
Alternative Strategy: US Data on Borrowers		
	Debt-to-Asset Ratio	
Active investors	Total Assets (vis-à-vis all sectors)	Total Assets (No household/public sec.)
Shadow banks[1]	0.351	0.293
Commercial banks	-	0.224
Commercial banks (no mutual funds shares)	-	0.221

Source: Institutional Investor Statistics, OECD database; Flow of Funds Accounts of the U.S., Federal Reserve Statistics.

[1] Shadow banks are finance companies, funding corporations, issuers of ABCP and security broker-dealers.

Table 3: Calibration for Equation (34) and it Foreign Counterpart

	Estimate (s.e.)	Criterion function	p-value OIR test	Moments to match			
				$\rho\left(c_t, Y_{Ht}\right)$	$\rho\left(q^e_{Ht}, Y_{Ht}\right)$	$Skew\left(r_{Ht}\right)$	$Kurt\left(r_{Ht}\right)$
data: $T = 35$	-	-	-	0.9241	0.7152	-0.7260	2.6880
SMM: $N = 10$	0.1233*** (0.0020)	0.1513	0.1204	0.6792	0.6236	0.0050	2.7683
SMM: $N = 20$	0.1215*** (0.0049)	0.1499	0.1381	0.7229	0.6783	-0.0556	2.7433
SMM: $N = 30$	0.1260*** (0.0034)	0.1488	0.1458	0.7021	0.6676	-0.0733	2.5614
Final Parametrization							
$\psi = 0.126$		$\rho_\kappa = 0.37$		$\sigma_\kappa = 0.03$		$\rho\left(\varepsilon_{\kappa t}, \varepsilon_{\kappa^* t}\right) = 0.64$	

*** indicates a confidence level $p < 0.001$.

Table 4: Equilibrium Portfolios

	Home		Foreign		Wealth
	Equity	Bond	Equity	Bond	
Investors					
Home passive	0.80	0.78	-	0.21	1.79
Home active	1.88	-0.78	1.30	-0.21	2.19
Foreign passive	-	0.21	0.80	0.78	1.79
Foreign active	1.30	-0.21	1.88	-0.78	2.19
Market clearing	3.98	0	3.98	0	7.96
Sectors					
Home backyard	0.80	0.99	-	-	1.79
Home traded good	3.18	-0.99	-	-	2.19
Foreign backyard	-	-	0.80	0.99	1.79
Foreign traded goods	-	-	3.18	-0.99	2.19

Note: The equilibrium asset holdings reported in the table are computed in value terms and as a share of economy-wide output. This equals $\overline{Y} + (1-n)\,z\left(\overline{k}^P\right)^{\nu}$.

Table 5: Portfolios: Home Bias and the Ensuing Effect on Counterparty Risk

	Parameter(s)			Home bias %		Effect of Collateral on κ_t	
				Equity	Funding	Local Col.	Foreign Col.
Baseline Calibration	-			59.1	78.8	0.213	0.450
Sensitivity To Increasing Model Parameters							
Preference Parameters	$\gamma, \theta, \gamma_I, \theta_I$			$\uparrow\downarrow$	$\uparrow$	-	-
Greater Leverage		$\bar{\kappa}\uparrow$	$\psi/\bar{\kappa}$				
	a)	0.41	const	59.1	92.4	0.2819	0.5953
	b)	0.41	$\uparrow$	59.1	92.4	0.3383	0.5137
	c)	0.41	$\downarrow$	59.1	92.4	0.2131	0.6947

Note: In terms of the results in Table 4, the definitions used are:
1) local bias in $j = \frac{\text{H (F) active investors' ownership of H (F) asset } j}{\text{stock of asset } i \text{ used by H (F) final goods sector}}$, where $j =$ equity, bond; 2) pure effect of riskiness of local collateral on counterparty risk $= \frac{\psi}{\text{local equity bias}}$; 3) pure effect of riskiness of foreign collateral on counterparty risk $= \frac{\psi^*}{\text{1 - local equity bias}}$.

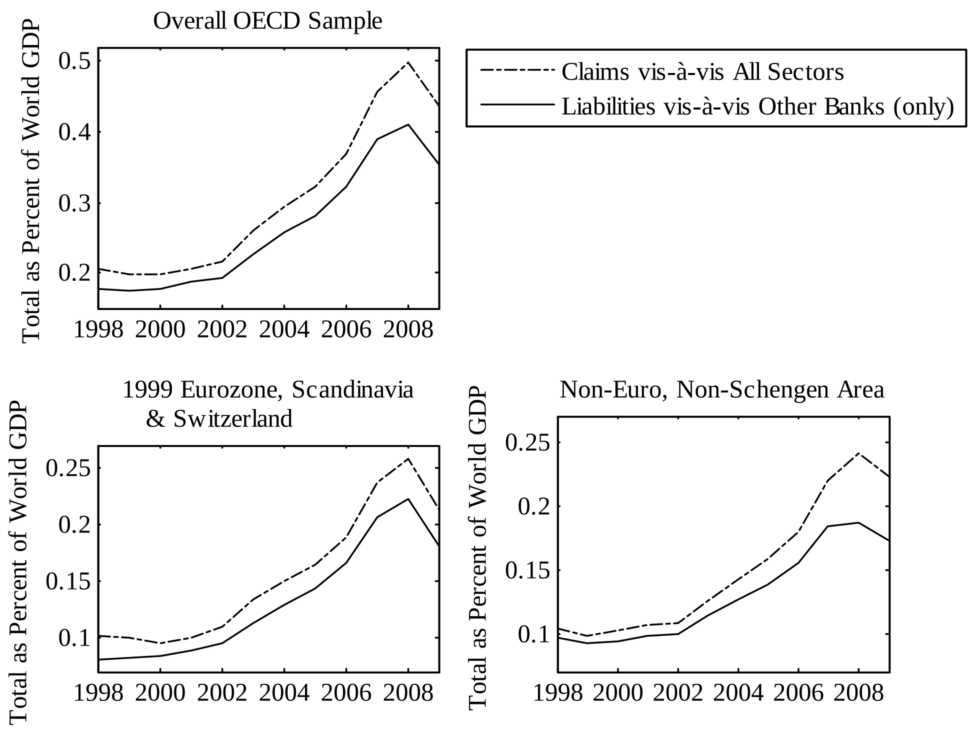

Figure 1: External Positions of Banks Operating in OECD Countries

Source: BIS Locational banking statistics; The World Bank WDI

Note: The sample of OECD countries considered involves Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the U.K. and the U.S.

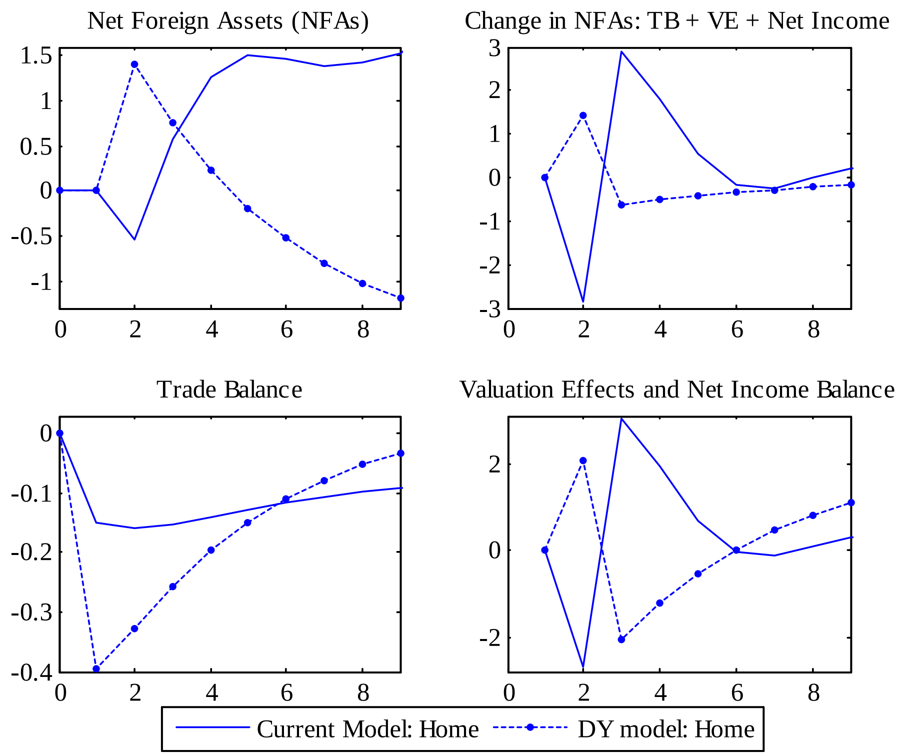

Figure 2: Current Model vs Devereux-Yetman: Predictions for (Net) IIP

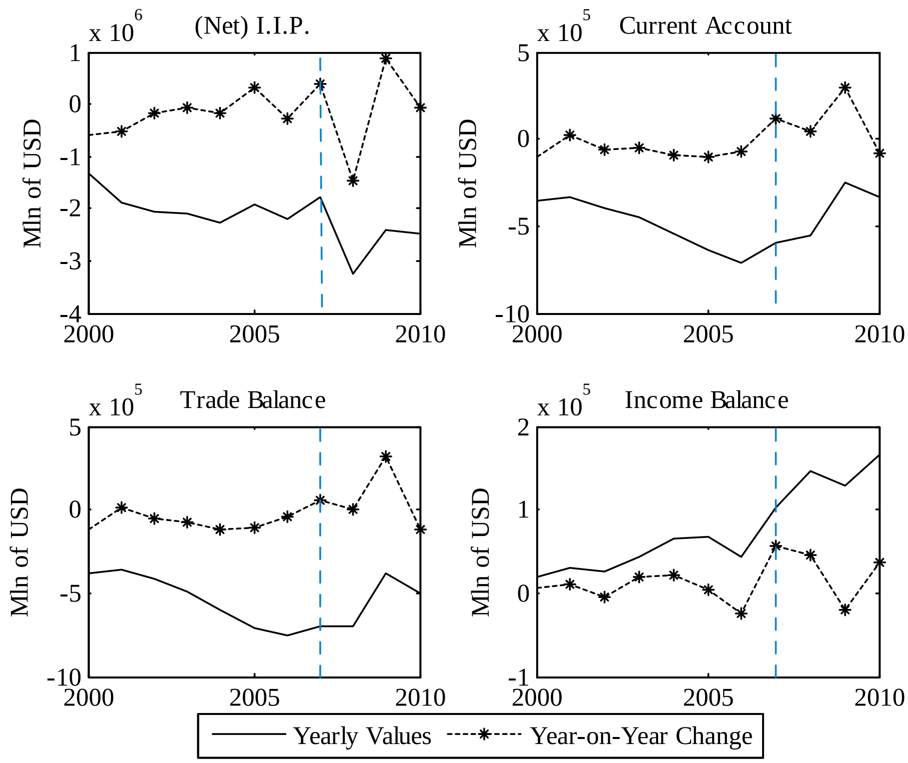

Figure 3: U.S. International Accounts in 2000-2010

Source: International Investment Position & International Transactions, U.S. Dept. of Commerce, Bureau of Economic Analysis.

Note: The vertical blue and dashed line (long dashes) refers to the year 2007.

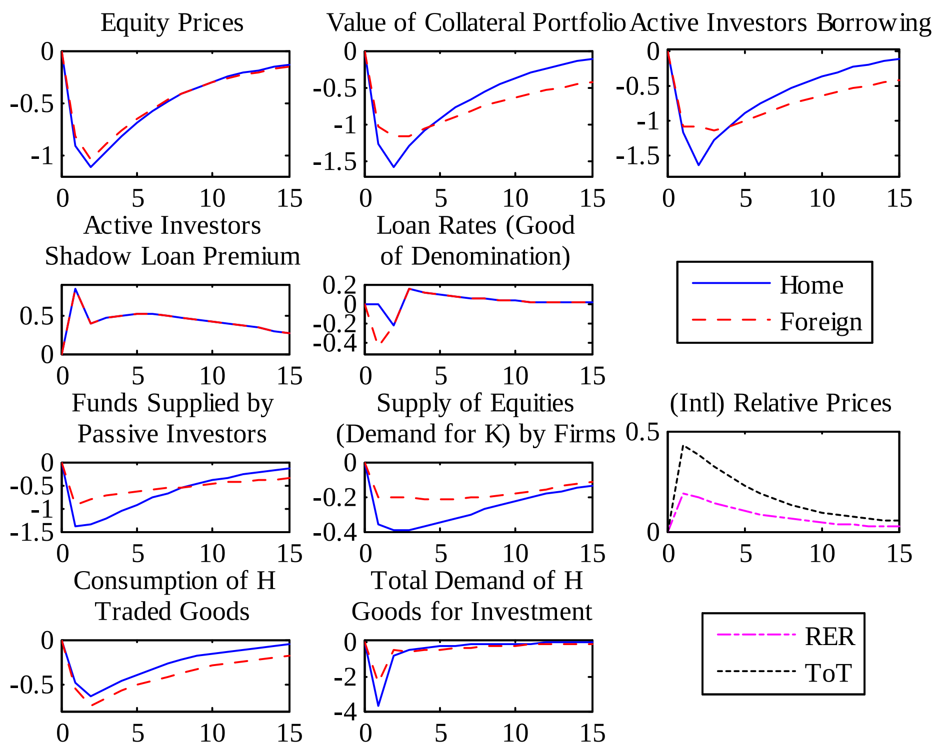

Figure 4: Debt Markets Integration: Productivity Shocks at Home

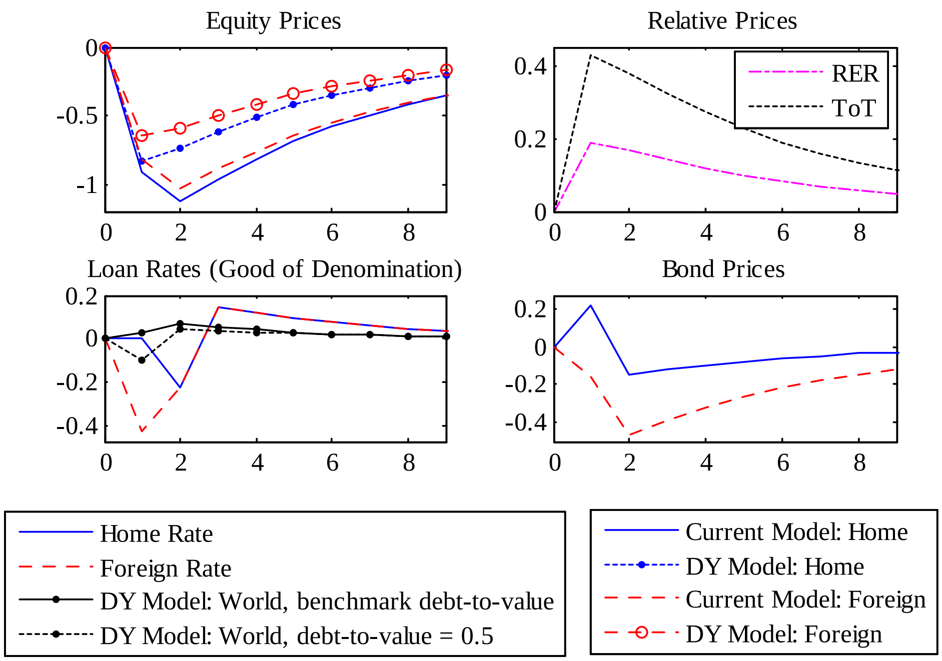

Figure 5: Debt Markets Integration: Price Effects

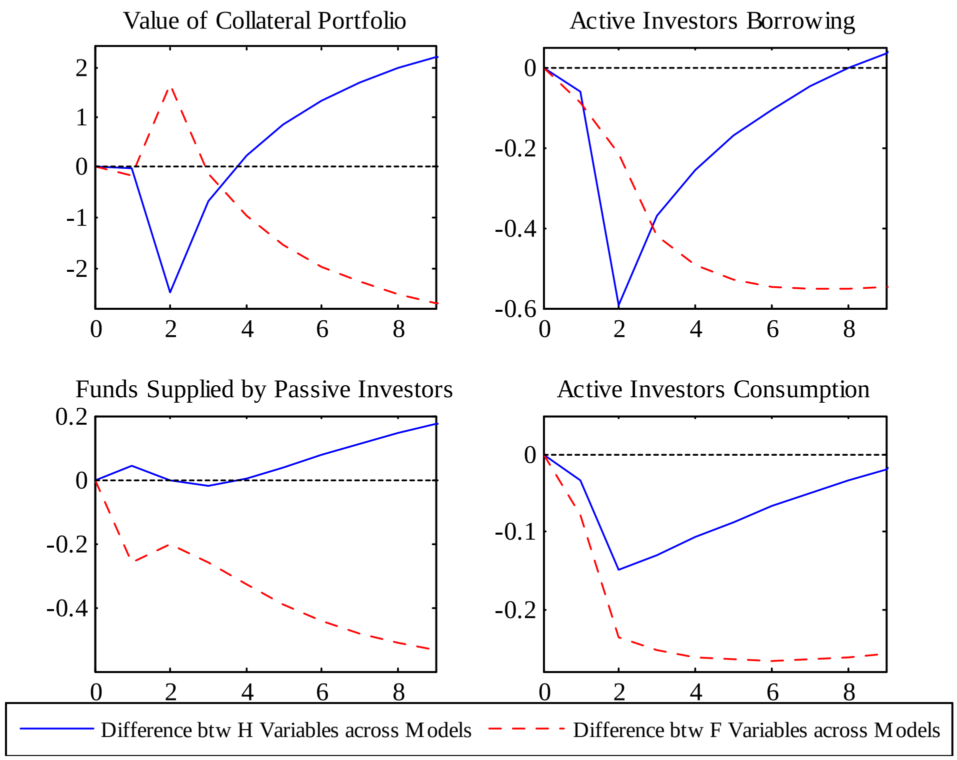

Figure 6: Debt Markets Integration: "Differential" Quantity Effects w.r.t. Devereux-Yetman

Note: The "Difference" is positive if the (home/foreign) variable considered reacts by more in Devereux and Yetman (2010) than here. A negative "Difference" can be interpreted in the opposite way.

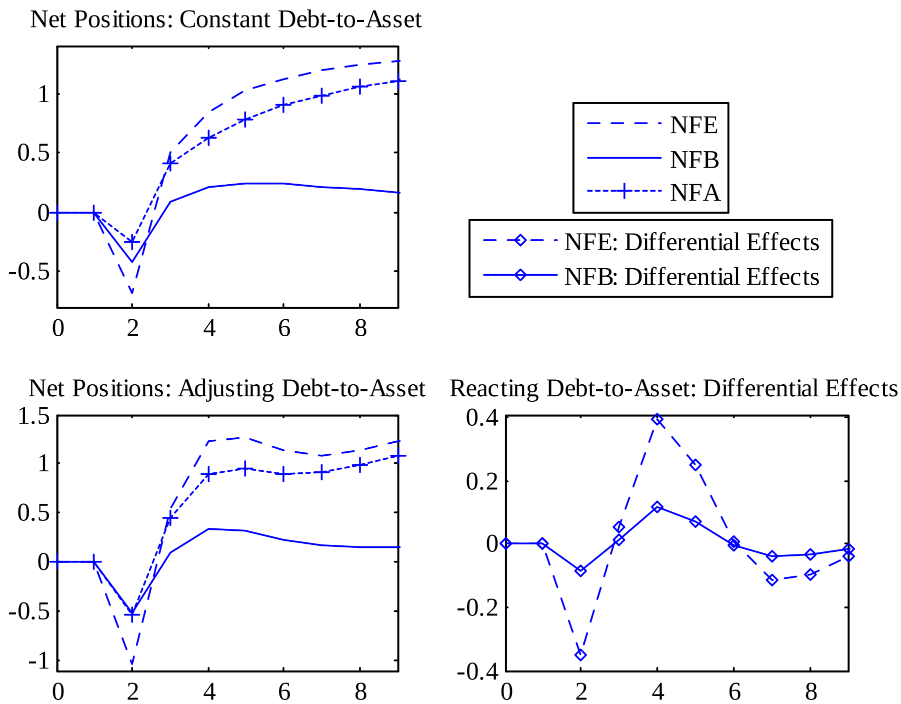

Figure 7: Debt Markets Integration: Home Country NFAs after Productivity Shocks

Note: The "Differential Effects" in the third panel are computed subtracting the impulse responses with constant debt-to-asset ratios from those with time-varying ratios.

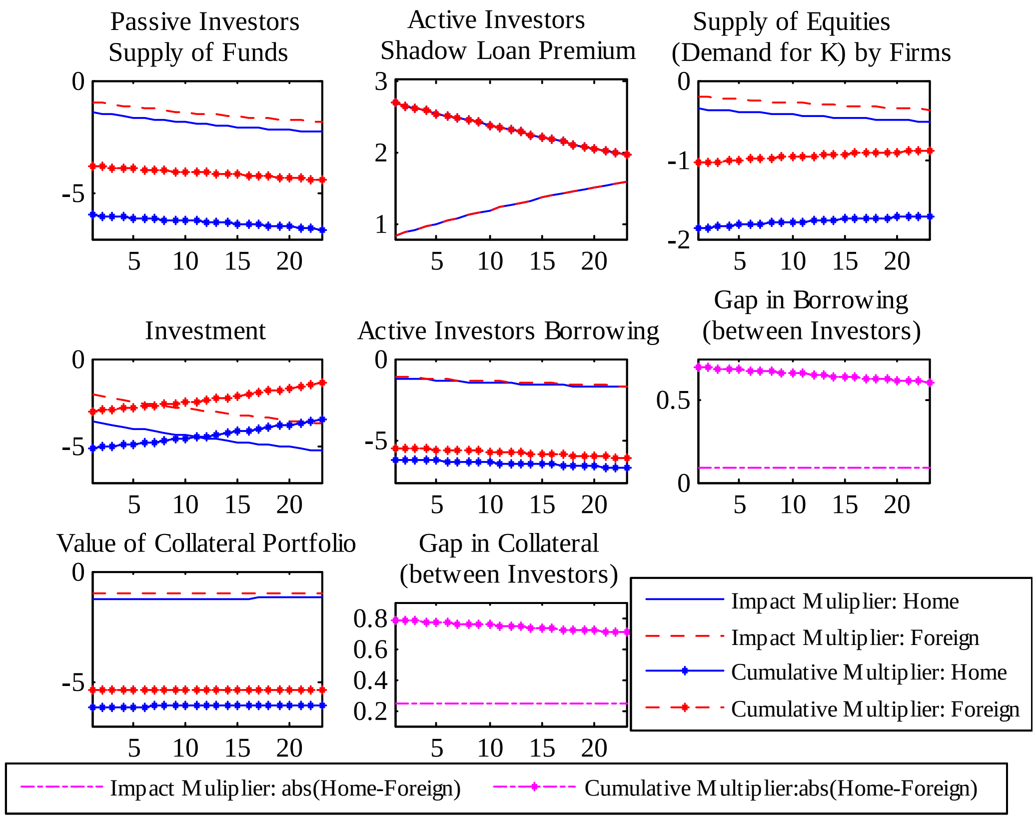

Figure 8: Effect of Adjusting in Haircuts: Impact vs Cumulative Multipliers

Note: ψ, ψ^* simultaneously increase in their respective ranges, with 23 steps each: $\psi \in [0{,}0.126]$, $\psi^* \in [0{,}0.185]$. The number of steps is on the x-axis.

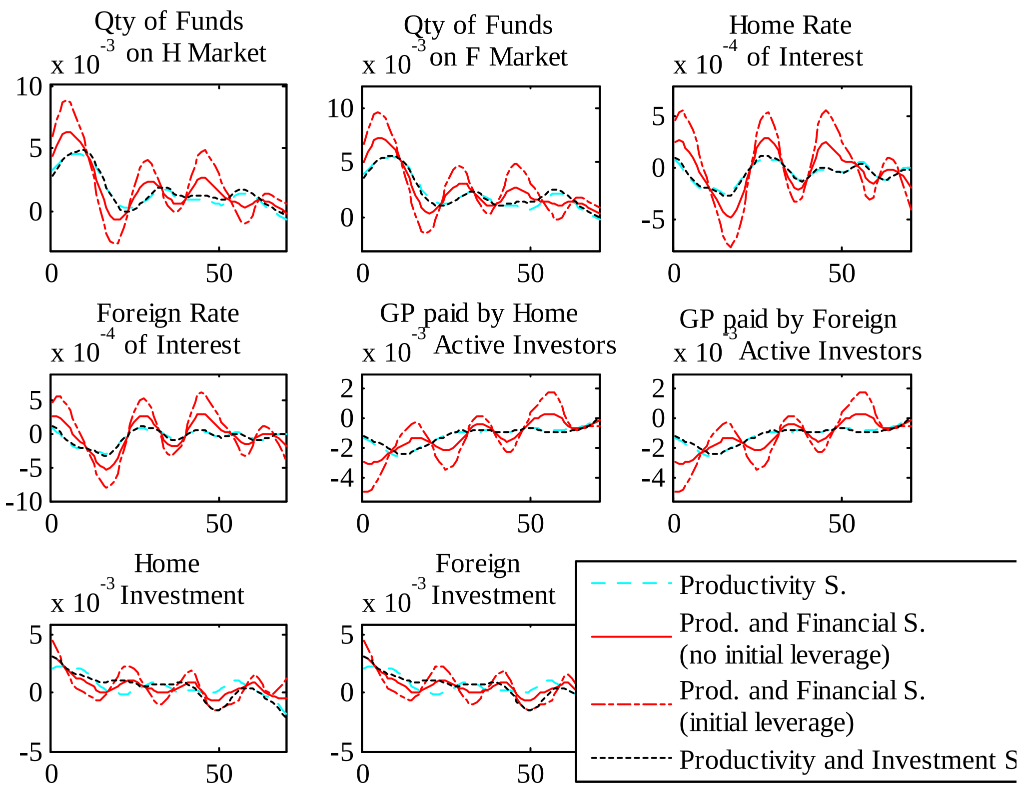

Figure 9: Simulations with Business Cycle and Financial Shocks: Effects on Volatility

A Appendix

A.1 Borrowing Limits and VaR

According to a widely used regulatory and risk management rule, a leveraged investor must cope with a value-at-risk (VaR) constraint or a constraint on the total margin on her positions. The finance literature shows that this type of constraint implies that borrowers face an haircut on each asset pledged as collateral.

VaR constraints are at work also in the present model. Adding the total value of equity portfolios to both sides of collateral constraints, equations (13) are rewritten as follows

$$m_t \left(q^e_{Ht} k^A_{Ht} + q^e_{Ft} k^A_{Ft} \right) \leq W^A_t \quad ; \quad m^*_t \left(q^e_{Ht} k^{*A}_{Ht} + q^e_{Ft} k^{*A}_{Ft} \right) \leq W^{*A}_t \tag{A.1}$$

where $m_t = 1 - \kappa_t$ and $m^*_t = 1 - \kappa^*_t$ are *home and foreign borrowers' haircuts*, and W^A_t, W^{*A}_t denote their (net) financial wealth. W^A_t is given by the left hand side of the budget constraint (12), except for consumption $P_t c_t^A$. So, in terms of net wealth, home active investors' budget constraint can be, equivalently, written as follows

$$P_t c_t^A + W_t^A = w_t + W_{t-1}^A + \sum_{i=H}^{F} \left[\left(q_{it}^e + d_{it} - q_{it-1}^e \right) k_{it-1}^A - \left(p_{it} - q_{it-1}^b \right) b_{it-1}^A \right] \text{ with } p_{Ht} = 1 \; \forall t$$

For foreign investors, the budget constraint can be written in a similar way.

There is an issue in that the haircuts in (A.1) are specific to the borrower considered, while the finance literature accounts for the riskiness of each collateral asset. Let $m_{it} = 1 - \kappa_{it}$ be the haircut imposed on asset i, with κ_{it} be the corresponding debt-to-asset ratio, with $i = H, F$. Along the lines of Brunnermeier and Pedersen (2009), binding total margins constraints imply that margin i must be set so that

$$\varsigma = \Pr \left[-\Delta q_{it+1}^e > m_{it} \right] \tag{A.2}$$

is small enough[30]. Given the link between the collateral constraint (13) and the margin constraints (A.1), also this model embeds a condition such as (A.2). Due to the endogenous portfolio choice, the agent-specific haircuts m_t, m_t^* are functions of the asset-specific haircuts m_{Ht}, m_{Ft}, depending on the equilibrium portfolio shares. Formally:

$$\begin{aligned} \varsigma &\approx \Pr \left[-\left(\frac{q_{Ht+1}^e}{q_{Ht}^e} \frac{q_{Ht} k_{Ht}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} + \frac{q_{Ft+1}^e}{q_{Ft}^e} \frac{q_{Ft} k_{Ft}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} \right) > m_t \right] \\ &= \Pr \left[-\left(\frac{q_{Ht+1}^e}{q_{Ht}^e} \frac{q_{Ht} k_{Ht}^{*A}}{q_{Ht} k_{Ht}^{*A} + q_{Ft} k_{Ft}^{*A}} + \frac{q_{Ft+1}^e}{q_{Ft}^e} \frac{q_{Ft} k_{Ft}^{*A}}{q_{Ht} k_{Ht}^{*A} + q_{Ft} k_{Ft}^{*A}} \right) > m_t^* \right] \end{aligned} \tag{A.3}$$

where I replace Δq_{it+1}^e with q_{it+1}^e / q_{it}^e, for modeling convenience. So home borrowers' haircut is

$$m_t = \frac{q_{Ht} k_{Ht}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} m_{Ht} + \frac{q_{Ft} k_{Ft}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} m_{Ft}$$

and m_t^* is specified in a similar way.

Given (A.2) and assuming that the distribution of asset returns is Normal, one generally finds that m_{it} is a function of the volatility of the current value of asset i. And this function is furthermore linear if the value of asset i follows a GARCH process.

But since in the main text I use collateral constraints instead of VaR constraints, I take advantage of the definition of m_t (and m_t^*) and of m_{Ht}, m_{Ft} to adapt these results to borrowers' debt-to-value ratios. I thus write

$$\kappa_t = \frac{q_{Ht} k_{Ht}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} \kappa_{Ht} + \frac{q_{Ft} k_{Ft}^A}{q_{Ht} k_{Ht}^A + q_{Ft} k_{Ft}^A} \kappa_{Ft}$$

which is equation (33); along the same lines, I also specify κ_t^*. Next, I capture (A.2) with the following assumption:

$$\kappa_{it} = f \left(\frac{q_{it}^e}{q_{it-1}^e} \right) \text{ with } i = H, F \;,\; f'(\cdot) > 0 \tag{A.4}$$

[30] The negative sign is used to characterize an unfavourable state of the world.

According to (A.2), once the confidence level ς has been chosen, financiers ask for a higher margin m_{it}, the greater is the expected fall in the market price of asset i. Since κ_{it} is inversely related to m_{it}, equation (A.4) indicates that the (percentage) fall in the price triggers a fall in the debt-to-asset ratio.

Note that my approach does not aim to capture a key step of Brunnermeier and Pedersen's analysis. In their model, the dynamic properties of the haircuts depends on financiers' information set, distinguishing between those that are informed about fundamental and liquidity risks and those that are not. This is not my goal here, so I do not make any assumption on multiple information sets. But this does not mean that financiers' expectations do not play any role. Important is indeed the interaction between lenders' and borrowers' expectations, whose properties are typical of RE model with heterogeneous agents. Active and passive investors' Euler equations in both countries are linked because of equilibrium equity pricing. See equations (47)-(48).

A.2 Conditions for Computing Portolios

Consider the steady state of the economy (denoted by a "bar" above the variables). Given equations (20)-(22), the loan rate is $\bar{R} = \beta\left(\bar{c}^P\right)^{-1}$. Using it in (14)-(15), the guarantee premium on loans is

$$\overline{GP} \equiv \frac{\bar{\mu}}{\beta\left(\bar{c}^A\right)\left(\bar{c}^A\right)^{-\sigma}} = \frac{\beta\left(\bar{c}^P\right) - \beta\left(\bar{c}^A\right)}{\beta\left(\bar{c}^A\right)\beta\left(\bar{c}^P\right)}$$

So, from (16)-(17), the rate of return on equity is

$$\bar{r} = \frac{1}{\beta\left(\bar{c}^A\right)}\left(1 - \bar{\kappa}\frac{\beta\left(\bar{c}^P\right) - \beta\left(\bar{c}^A\right)}{\beta\left(\bar{c}^P\right)}\right)$$

Hence, equilibrium portfolios can be computed conditionally on the following two steady state features: the first is useful for computing the equity portfolio, the second is consequential to the first and is useful to determine the bond portfolio.

Condition 1. As shown by Devereux and Yetman (2010), the ownership of the stock of capital used by firms ($\bar{K} = n\bar{\chi}$) is diversified across countries depending on how the total capital available in each (symmetric) economy, $\bar{K} + (1-n)\,\bar{k}^P$, is allocated across the productive sectors of that economy. In fact, in the steady state, (14)-(17), (20)-(23) and (27) determine $\bar{\chi}$ and $\bar{k}^P$ as a result of the following system:

$$\begin{aligned}
\frac{1}{1-\beta\left(\bar{c}^P\right)}\nu z\left(\bar{k}^P\right)^{\nu-1} &= \frac{\beta\left(\bar{c}^A\right)}{1-\beta\left(\bar{c}^A\right) - \bar{\kappa}\bar{\mu}/\left(\bar{c}^A\right)^{-\sigma}}\bar{d} \\
n\bar{\chi} + (1-n)\,\bar{k}^P &= 1
\end{aligned}$$

with $\bar{\chi} = \bar{k}^A + \bar{k}^{*A}$.

Under binding constraints, one can combine the value obtained for $\bar{\chi}$ with (13), in order to determined the equilibrium value of active investors' collateral.

Condition 2. Between active investors, the total per-capita amount of collateralized debt is "in positive net supply". In fact, the total amount of loans that financiers in each economy agree to purchase is $\bar{B}^P = -n\bar{B}^A/(1-n)$. And from (13), $\bar{B}^A$ is

$$\bar{B}^A = \frac{\bar{\kappa}\bar{q}^e\bar{\chi}}{\bar{q}^b}$$

with $\bar{B}^A = \bar{b}^A + \bar{b}^{*A}$.

A.3 Additional Graphs

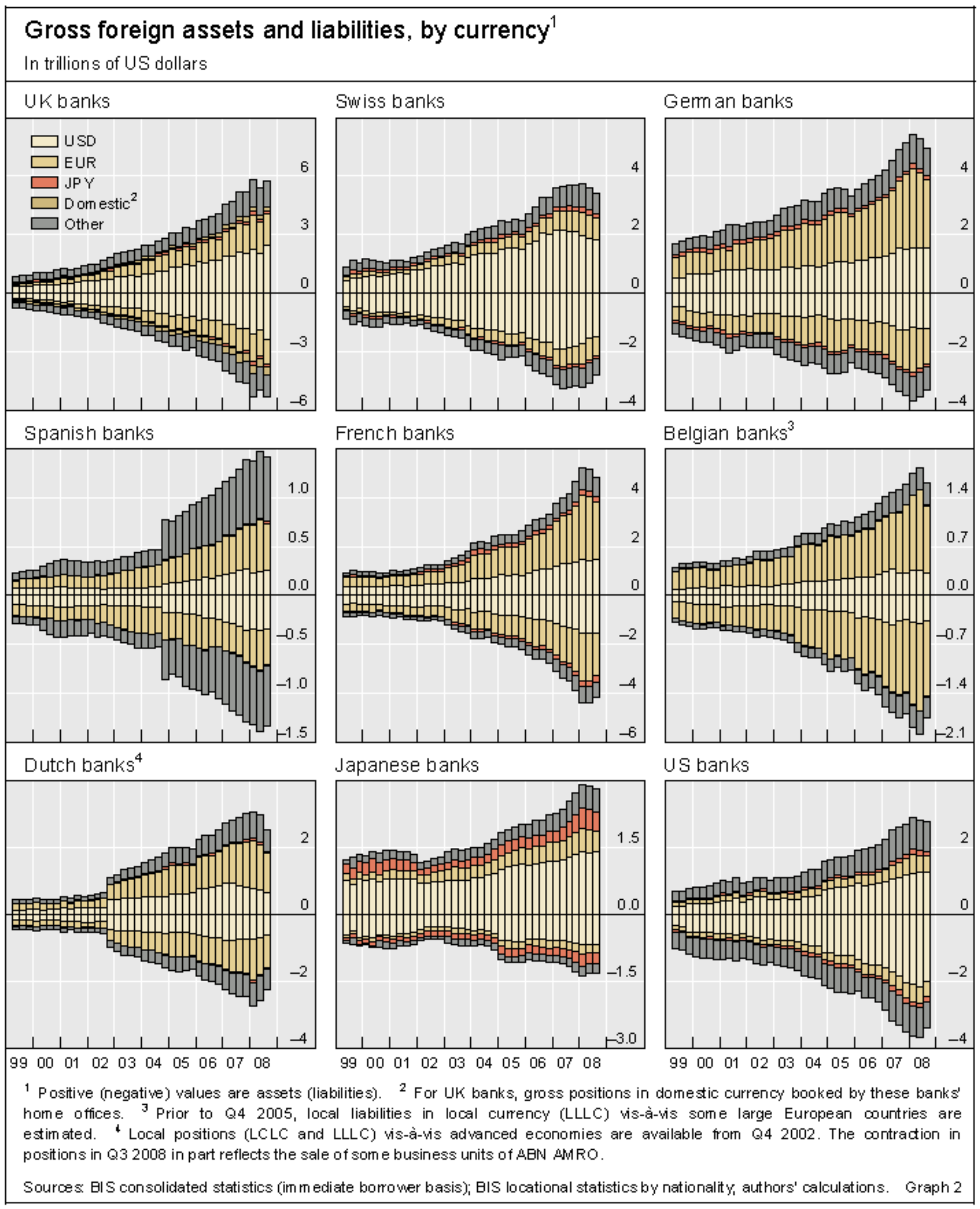

Figure 1-A: Currency Breakdown of External Bank Positions

Source: Taken from McGuire and Von Peter (2009), p. 51, Graph 2.

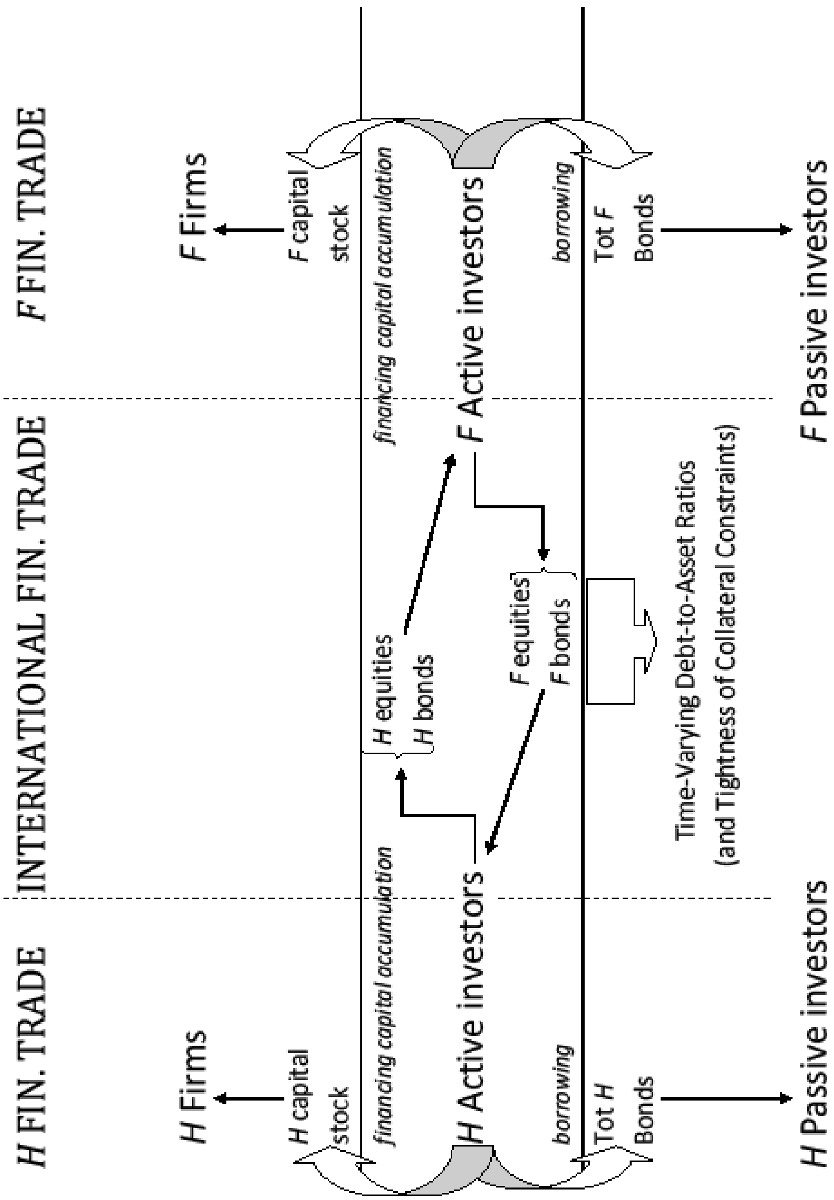

Figure 2-A: Diagram of the Financial Transactions in the Model

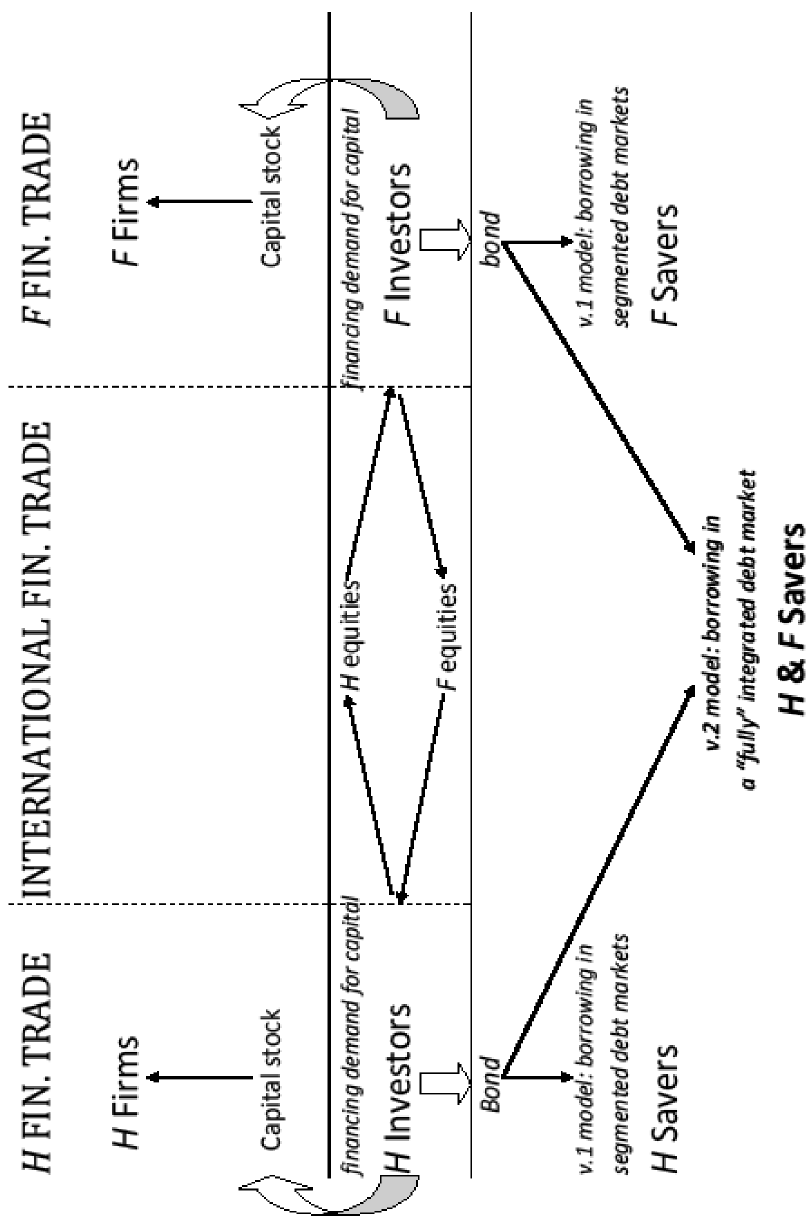

Figure 3-A: Diagram of the Financial Transactions in Devereux and Yetman (2010)

Funding under Borrowing Limits in International Portfolios
Separate Technical Appendix

Tommaso Trani
The Graduate Institute | Geneva

Contents

1 General Equilibrium

1.1 Home Country

- Firms

$$
\begin{aligned}
n\chi_{Ht} &= (1-\delta)\, n\chi_{Ht-1} + \Xi_t I_{t-1} \\
P_t^I &= \left[\gamma_I + (1-\gamma_I)\, p_{Ft}^{1-\theta_I}\right]^{\frac{1}{1-\theta_I}} \\
I_{Ht} &= \gamma_I \left(\frac{1}{P_t^I}\right)^{-\theta_I} I_t \\
I_{Ft} &= (1-\gamma_I) \left(\frac{p_{Ft}}{P_t^I}\right)^{-\theta_I} I_t \\
d_{Ht} &= \alpha A_t \left(n\chi_{Ht-1}^A\right)^{\alpha-1} - \frac{P_t^I I_{t-1}}{n\chi_{Ht-1}} \\
w_t &= (1-\alpha)\, A_t \left(n\chi_{Ht-1}\right)^{\alpha}
\end{aligned}
$$

- Allocation of consumption between goods

$$
\begin{aligned}
P_t &= \left[\gamma + (1-\gamma)\, p_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \implies \pi_t = \frac{P_t}{P_{t-1}} \\
c_{Ht}^h &= \gamma \left(\frac{1}{P_t}\right)^{-\theta} c_t^h \text{ for } h = A, P \\
c_{Ft}^h &= (1-\gamma) \left(\frac{p_{Ft}}{P_t}\right)^{-\theta} c_t^h \text{ for } h = A, P
\end{aligned}
$$

- Active Investors

$$\left[\left(c_t^A\right)^{-\sigma} - \mu_t P_t\right] q_{Ht}^b = \zeta^A \left(1 + c_t^A\right)^{-\phi} E_t \left(c_{t+1}^A\right)^{-\sigma} \frac{1}{\pi_{t+1}} \tag{1}$$

$$\left[\left(c_t^A\right)^{-\sigma} - \mu_t P_t\right] q_{Ft}^b = \zeta^A \left(1 + c_t^A\right)^{-\phi} E_t \left(c_{t+1}^A\right)^{-\sigma} \frac{p_{Ft+1}}{\pi_{t+1}} \tag{2}$$

$$\left[\left(c_t^A\right)^{-\sigma} - \mu_t P_t \kappa_t\right] q_{Ht}^e = \zeta^A \left(1 + c_t^A\right)^{-\phi} E_t \left(c_{t+1}^A\right)^{-\sigma} \frac{\left(q_{Ht+1}^e + d_{Ht+1}\right)}{\pi_{t+1}} \tag{3}$$

$$\left[\left(c_t^A\right)^{-\sigma} - \mu_t P_t \kappa_t\right] q_{Ft}^e = \zeta^A \left(1 + c_t^A\right)^{-\phi} E_t \left(c_{t+1}^A\right)^{-\sigma} \frac{\left(q_{Ft+1}^e + d_{Ft+1}\right)}{\pi_{t+1}} \tag{4}$$

$$
\begin{aligned}
P_t c_t^A - q_{Ht}^b b_{Ht}^A - q_{Ft}^b b_{Ft}^A + q_{Ht}^e k_{Ht}^A + q_{Ft}^e k_{Ft}^A &= w_t - b_{Ht-1}^A - p_{Ft} b_{Ft-1}^A \\
&\quad + \left(q_{Ht}^e + d_{Ht}\right) k_{Ht-1}^A + \left(q_{Ft}^e + d_{Ft}\right) k_{Ft-1}^A \qquad (5) \\
q_{Ht}^b b_{Ht}^A + q_{Ft}^b b_{Ft}^A &\leq \kappa_t \left(q_{Ht}^e k_{Ht}^A + q_{Ft}^e k_{Ft}^A\right) \qquad (6) \\
\kappa_t &= \psi \frac{q_{Ht}^e}{q_{Ht-1}^e} + \psi^* \frac{q_{Ft}^e}{q_{Ft-1}^e} + \frac{\bar{m}}{\bar{\kappa}} \epsilon_{\kappa t} \qquad (7)
\end{aligned}
$$

- Passive Investors

$$\begin{aligned}
\left(c_t^P\right)^{-\sigma} q_{Ht}^b &= \zeta^P \left(1+c_t^P\right)^{-\phi} E_t \left(c_{t+1}^P\right)^{-\sigma} \frac{1}{\pi_{t+1}} \\
\left(c_t^P\right)^{-\sigma} q_{Ht}^e &= \zeta^P \left(1+c_t^P\right)^{-\phi} E_t \left(c_{t+1}^P\right)^{-\sigma} \frac{\left[q_{Ht+1}^e + \nu z \left(k_{Ht}^P\right)^{\nu-1}\right]}{\pi_{t+1}} \\
P_t c_t^P + q_{Ht}^e \left(k_{Ht}^P - k_{Ht-1}^P\right) - q_{Ht}^b B_{Ht}^P &= w_t^P + z \left(k_{Ht-1}^P\right)^{\nu} - B_{Ht-1}^P
\end{aligned}$$

- Market Clearing

$$\begin{aligned}
n\left(c_{Ht}^A + c_{Ht}^{*A}\right) + I_{Ht} + I_{Ht}^* + (1-n)\left(c_{Ht}^P + c_{Ht}^{*P}\right) &= A_{Ht}\left(n\chi_{Ht}\right)^{\alpha} + (1-n)\, z \left(k_{Ht-1}^P\right)^{\nu} \\
n\left(b_{Ht}^A + b_{Ht}^{*A}\right) + (1-n)\, B_{Ht}^P &= 0 \qquad (8) \\
n\chi_{Ht}^A + (1-n)\, k_{Ht}^P &= 1
\end{aligned}$$

1.2 Foreign Country

- Firms

$$\begin{aligned}
n\chi_{Ft}^A &= (1-\delta)\, n\chi_{Ft-1}^A + \Xi_t^* I_{t-1}^* \\
P_t^{*I} &= \left[(1-\gamma_I) + \gamma_I p_{Ft}^{1-\theta_I}\right]^{\frac{1}{1-\theta_I}} \\
I_{Ht}^* &= (1-\gamma_I)\left(\frac{1}{P_t^{*I}}\right)^{-\theta_I} I_t^* \\
I_{Ft}^* &= \gamma_I \left(\frac{p_{Ft}}{P_t^{*I}}\right)^{-\theta_I} I_t^* \\
d_{Ft} &= p_{Ft}\alpha A_t^* \left(n\chi_{Ft-1}\right)^{\alpha-1} - \frac{P_t^{*I} I_{t-1}^*}{n\chi_{Ft-1}} \\
w_t^* &= p_{Ft}\,(1-\alpha)\, A_t^* \left(n\chi_{Ft-1}\right)^{\alpha}
\end{aligned}$$

- Allocation of consumption between goods

$$\begin{aligned}
P_t^* &= \left[(1-\gamma) + \gamma p_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \implies \pi_t^* = \frac{P_t^*}{P_{t-1}^*} \\
c_{Ht}^{*h} &= (1-\gamma)\left(\frac{1}{P_t^*}\right)^{-\theta} c_t^{*h} \quad \text{for } h = A, P \\
c_{Ft}^{*h} &= \gamma \left(\frac{p_{Ft}}{P_t^*}\right)^{-\theta} c_t^{*h} \quad \text{for } h = A, P
\end{aligned}$$

- Active Investors

$$\left[\left(c_t^{*A}\right)^{-\sigma} - \mu_t^* P_t^*\right] q_{Ht}^b = \zeta^A \left(1 + c_t^{*A}\right)^{-\phi} E_t \left(c_{t+1}^{*A}\right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \tag{9}$$

$$\left[\left(c_t^{*A}\right)^{-\sigma} - \mu_t^* P_t^*\right] q_{Ft}^b = \zeta^A \left(1 + c_t^{*A}\right)^{-\phi} E_t \left(c_{t+1}^{*A}\right)^{-\sigma} \frac{p_{Ft+1}}{\pi_{t+1}^*} \tag{10}$$

$$\left[\left(c_t^{*A}\right)^{-\sigma} - \mu_t^* P_t^* \kappa_t^*\right] q_{Ht}^e = \zeta^A \left(1 + c_t^{*A}\right)^{-\phi} E_t \left(c_{t+1}^{*A}\right)^{-\sigma} \frac{\left(q_{Ht+1}^e + d_{Ht+1}\right)}{\pi_{t+1}^*} \tag{11}$$

$$\left[\left(c_t^{*A}\right)^{-\sigma} - \mu_t^* P_t^* \kappa_t^*\right] q_{Ft}^e = \zeta^A \left(1 + c_t^{*A}\right)^{-\phi} E_t \left(c_{t+1}^{*A}\right)^{-\sigma} \frac{\left(q_{Ft+1}^e + d_{Ft+1}\right)}{\pi_{t+1}^*} \tag{12}$$

$$\begin{aligned} P_t c_t^{*A} - q_{Ht}^b b_{Ht}^{*A} - q_{Ft}^b b_{Ft}^{*A} + q_{Ht}^e k_{Ht}^{*A} + q_{Ft}^e k_{Ft}^{*A} &= w_t^* - b_{Ht-1}^{*A} - p_{Ft} b_{Ft-1}^{*A} \\ &\quad + \left(q_{Ht}^e + d_{Ht}\right) k_{Ht-1}^{*A} + \left(q_{Ft}^e + d_{Ft}\right) k_{Ft-1}^{*A} \end{aligned} \tag{13}$$

$$q_{Ht}^b b_{Ht}^{*A} + q_{Ft}^b b_{Ft}^{*A} \leq \kappa_t^* \left(q_{Ht}^e k_{Ht}^{*A} + q_{Ft}^e k_{Ft}^{*A}\right) \tag{14}$$

$$\kappa_t^* = \psi^* \frac{q_{Ht}^e}{q_{Ht-1}^e} + \psi \frac{q_{Ft}^e}{q_{Ft-1}^e} + \frac{\bar{m}}{\bar{\kappa}} \epsilon_{\kappa^* t} \tag{15}$$

- Passive Investors

$$\left(c_t^{*P}\right)^{-\sigma} q_{Ft}^b = \zeta^P \left(1 + c_t^{*P}\right)^{-\phi} E_t \left(c_{t+1}^{*P}\right)^{-\sigma} \frac{p_{Ft+1}}{\pi_{t+1}^*}$$

$$\left(c_t^{*P}\right)^{-\sigma} q_{Ft}^e = \zeta^P \left(1 + c_t^{*P}\right)^{-\phi} E_t \left(c_{t+1}^{*P}\right)^{-\sigma} \frac{\left[q_{Ft+1}^e + p_{Ft+1} \nu z \left(k_{Ft}^{*P}\right)^{\nu-1}\right]}{\pi_{t+1}^*}$$

$$P_t^* c_t^{*P} + q_{Ft}^e \left(k_{Ft}^{*P} - k_{Ft-1}^{*P}\right) - q_{Ft}^b B_{Ft}^{*P} = w_t^{*P} + p_{Ft} \left[z \left(k_{Ft-1}^{*P}\right)^{\nu} - B_{Ft-1}^P\right]$$

- Market Clearing

$$\begin{aligned} n\left(c_{Ft}^A + c_{Ft}^{*A}\right) + I_{Ft} + I_{Ft}^* + (1-n)\left(c_{Ft}^P + c_{Ft}^{*P}\right) &= p_{Ft}\left[A_{Ft}\left(n\chi_{Ft}\right)^{\alpha} + (1-n) z \left(k_{Ft-1}^{*P}\right)^{\nu}\right] \\ n\left(b_{Ft}^A + b_{Ft}^{*A}\right) + (1-n) B_{Ft}^P &= 0 \\ n\chi_{Ft} + (1-n) k_{Ft}^{*P} &= 1 \end{aligned} \tag{16}$$

2 Portfolio Choice Problem

2.1 Non-Portfolio Equations

I solve for portfolios in the way described by Devereux and Surtherland (2011).

As shown in the text, the effect of international portfolios on the behaviour of model variables shows up when equations (5)-(6), (8), (13)-(14) and (16) are re-specified in terms of

$$\begin{aligned} B_{it}^A &= b_{it}^A + b_{it}^{*A} \quad \text{with} \quad i = H, F & (17) \\ NFE_t^A &= q_{Ft}^e k_{Ft}^A - q_{Ht}^e \left(\chi_{Ht} - k_{Ht}^A \right) & (18) \\ NFB_t^A &= q_{Ft}^b b_{Ft}^A - q_{Ht}^b \left(B_{Ht}^A - b_{Ht}^A \right) & (19) \end{aligned}$$

and clearly of the portfolio shares ω_t^e, ω_t^b for equities and bonds, respectively.

Budget constraints are the only equations that are directly affected by $\tilde{\omega}$. Consider then the budget constraint of home country active investors, re-specified as just said. Letting

$$\tilde{\omega} = \begin{bmatrix} \tilde{\omega}^b \\ \tilde{\omega}^e \end{bmatrix}$$

be the vector of equilibrium portfolio shares (a vector of constants), a first order approximation of the budget constraint yields

$$\begin{aligned} \hat{c}_t^A + \hat{P}_t + \left(\widehat{NFE}_t^A - \widehat{NFB}_t^A \right) &= \frac{\bar{w}_t^A}{\bar{c}^A} \hat{w}_t^A - \frac{\bar{q}^e \bar{\chi}^A}{\bar{c}^A} \hat{\chi}_{Ht}^A + \frac{\bar{r} \bar{q}^e \bar{\chi}^A}{\bar{c}^A} \hat{\chi}_{Ht-1}^A + \frac{\bar{d} \bar{\chi}^A}{\bar{c}^A} \hat{d}_{Ht} \\ &+ \frac{\bar{q}^b \bar{B}^A}{\bar{c}^A} \left(\hat{B}_{Ht}^A + \hat{q}_{Ht}^b \right) - \frac{\bar{B}^A}{\bar{c}^A} \hat{B}_{Ht-1} + \tilde{\omega}^e \hat{r}_{xt} - \tilde{\omega}^b \hat{R}_{xt} \\ &+ \bar{r} \widehat{NFE}_{t-1}^A - \bar{R} \widehat{NFB}_{t-1}^A \qquad (20) \end{aligned}$$

where $\widehat{NFE}_t^A = NFE^A / \bar{c}^A$, $\widehat{NFB}_t^A = NFB^A / \bar{c}^A$, $\hat{r}_{xt} = \hat{r}_{Ht} - \hat{r}_{Ft}$ and $\hat{R}_{xt} = \hat{R}_{Ht} - \hat{R}_{Ft}$.

2.2 Portfolio Equations

Applying Remark 1. in the text, the consumption Euler equations (1)-(4) and (9)-(12) yield two portfolio choice equations for both home and foreign active investors. In compact form, these are:

$$\begin{aligned} E_t \frac{\left(c_{t+1}^A \right)^{-\sigma}}{\pi_{t+1}} \begin{bmatrix} R_{Ht+1} - R_{Ft+1} \\ r_{Ht+1} - r_{Ft+1} \end{bmatrix} &= \underset{2 \times 1}{0} \\ E_t \frac{\left(c_{t+1}^{*A} \right)^{-\sigma}}{\pi_{t+1}^*} \begin{bmatrix} R_{Ht+1} - R_{Ft+1} \\ r_{Ht+1} - r_{Ft+1} \end{bmatrix} &= \underset{2 \times 1}{0} \end{aligned}$$

Taking a second order approximation of both of these portfolio Eulers, I find that the equilibrium portfolio $\tilde{\omega}$ satisfies

$$E_t \left[\hat{c}_{t+1}^A + \frac{\hat{\pi}_{t+1}}{\sigma} - \left(\hat{c}_{t+1}^{*A} + \frac{\hat{\pi}_{t+1}^*}{\sigma} \right) \right] \begin{bmatrix} \hat{R}_{xt+1} \\ \hat{r}_{xt+1} \end{bmatrix} = \underset{2 \times 1}{0} \qquad (21)$$

where the vector of asset returns differentials, $\begin{bmatrix} \hat{R}_{xt+1} & \hat{r}_{xt+1} \end{bmatrix}'$, is such that

$$E_t \begin{bmatrix} \hat{R}_{xt+1} \\ \hat{r}_{xt+1} \end{bmatrix} = -\frac{1}{2} E_t \begin{bmatrix} \hat{R}_{Ht+1}^2 - \hat{R}_{Ft+1}^2 \\ \hat{r}_{Ht+1}^2 - \hat{r}_{Ft+1}^2 \end{bmatrix} + \frac{1}{2} E_t \left[\sigma \left(\hat{c}_{t+1}^A + \hat{c}_{t+1}^{*A} \right) + \hat{\pi}_{t+1} + \hat{\pi}_{t+1}^* \right] \begin{bmatrix} \hat{R}_{xt+1} \\ \hat{r}_{xt+1} \end{bmatrix} \qquad (22)$$

2.3 Solution

Given the result in (22), $\begin{bmatrix} \hat{R}_{xt+1} & \hat{r}_{xt+1} \end{bmatrix}'$ does not depend on first order terms. So the linearized model can be solved treating $\begin{bmatrix} \hat{R}_{xt+1} & \hat{r}_{xt+1} \end{bmatrix}'$ as a wealth shock which equals zero in expectation. Let then

$$\boldsymbol{\xi}_t \equiv \begin{bmatrix} \xi_t^b \\ \xi_t^e \end{bmatrix} = \tilde{\omega}' \begin{bmatrix} \hat{R}_{xt+1} \\ \hat{r}_{xt+1} \end{bmatrix}$$

be a vector of shocks with these properties. Using the elements of $\boldsymbol{\xi}_t$ to replace the corresponding terms in (20), I can solve for model variables and for portfolios in the way described by Devereux and Sutherland (2011). First, I solve for model variables using a standard solution method for linear dynamic systems and adapting it for the effect of $\boldsymbol{\xi}_t$ on the budget constraint. Second, I extract from the entire solution of the model only the variables that enter into condition (21). Finally, I use these rows to compute the value of $\tilde{\omega}$ satifying that condition.

The result I obtain, together with a series of sensitivity tests, are in the following table.

		Home bias %		Effect of Collateral on κ_t	
	Parameter(s)	Equity	Funding	Local Col.	Foreign Col.
Baseline calibration	-	59.1	78.8	0.213	0.450
Sensitivity To Model Parameters					
Home bias in consumption	$\gamma \nearrow$ (to 0.82)	59.8	79.8	-	-
Substitutability in consumption	$\theta \nearrow$ (to 1.05)	56.9	94.9	-	-
Home bias in investment	$\gamma_I \nearrow$ (to 0.85)	69.0	88.9	-	-
Substitutability in investment	$\theta_I \nearrow$ (to 1.1)	58.9	79.8	-	-
Leverage	$\bar{\kappa} \nearrow$ $\psi/\bar{\kappa}$				
	a) 0.41 const	59.1	92.4	0.2819	0.5953
	b) 0.41 $\nearrow$	59.1	92.4	0.3383	0.5137
	c) 0.41 $\searrow$	59.1	92.4	0.2131	0.6947

Note: See legenda to Table 5 in the text.

3 SMM Estimation of ψ

I estimate the parameters governing the dynamics of κ_t, κ_t^* by SMM, drawing on the GARCH approach to Value-at-Risk (VaR)[1]. Indeed, it is standard for the studies on VaR or total margin constraints to assume that the fundamental value of assets follows a GARCH process (e.g., Brunnermeier and Pedersen, 2009). However, I also consider other approaches. For example, Iacoviello (2005) measures

[1] See, for instance, Tsay (2005).

the effects of loan-to-value ratios by matching impulse response functions. Michaelides and Ng (2000), estimating a RE pricing equation by SMM, find that the skewness and the kurtosis of prices improve the performance of the estimating procedure.

The setup is as follows. First, given (7), (15) and a value for $\bar{\kappa}$ (calibrated on observed features of passive investors' balance sheets), the parameters to estimate are ψ, ψ^*. Yet, $\bar{\kappa} = \psi + \psi^*$, so only one between ψ and ψ^* is identifiable. As shown below (Figure 1), it is ψ. Second, I proxy $\epsilon_{\kappa t}, \epsilon_{\kappa^* t}$ with the log-return on local stock market indices (which follow an AR(1) process). Finally, the moments I choose to match are: 1) the home consumption-output correlation; 2) the home stock price-output correlation; 3) the skewness of returns on stocks; 4) the kurtosis of returns on stocks.

Let M_t^d be the vector of these four moments computed on observed data and $M_t(\psi)$ the one based on model simulations. The solution of the model involves an endogenous international portfolio allocation, so $M_t(\psi)$ are in principle conditional on such an allocation. Put it differently, the simulated time-series are obtained only after the determination of the equilibrium portfolios. Following Duffie and Singleton (1993), these conditional simulations are used to estimate ψ as follows:

$$\begin{aligned}
G_T(\psi) &= \frac{1}{T}\sum_{t=1}^{T} M_t^d - \frac{1}{N}\sum_{t=1}^{NT} M_t(\psi) \\
W_T &= \left(1+\frac{1}{N}\right)\sum_{l=-\infty}^{\infty} E\left[\left(M_t^d - EM_t^d\right)\left(M_{t-l}^d - EM_{t-l}^d\right)'\right] \\
\psi^o &= \arg\min_{\psi\in\Psi} G_T(\psi)' W_T G_T(\psi) \\
\sigma_{\psi^o}^2 &= \left(1+\frac{1}{N}\right)\left(D' W_T^{-1} D\right)^{-1} \\
OIR &= \left(1+\frac{1}{N}\right) T G_T(\psi^o)' W_T G_T(\psi^o) \quad \sim \chi^2(3)
\end{aligned} \tag{23}$$

First, the optimal weighting matrix, W_T, coincides with the empirical covariance matrix, Σ_0^{-1}. Obviously, using the latter covariance matrix is an approximation, but this approximation is accurate as long as $N \to \infty$. In addition, Σ_0^{-1} is obtained with the Newey-West estimator. Second, the derivatives $D = E\partial M_t(\psi)/\partial\psi$ are computed numerically as the time-mean of two-sided differences. Finally, the estimated value (23) is found by grid serch (see below for details). The caveat is that the numerical approximation of D is not very precise: grid search is a "derivative-free" optimization. This problem is bigger, the smaller the grid. So I expand my grid as much as possible (unfortunately, losing in terms of computational speed).

3.1 Grid Search Optimization

One possible source of complication in designing an SMM estimation is that here the solution of the linearized model pins down the behaviour of model variables which is useful to compute the equilibrium value of country portfolios. The most convenient optimization procedures to accomodate an SMM estimation within such a framework are grid search and ascent gradient. But since in the present context the latter does not perform better than the former and entail much more care, I have eventually opted for grid search.

Intuitively, the grid search is good enough for my purpose, because the estimation of ψ or ψ^* is simplified by the fact that $\psi + \psi^* \in [0, \bar{\kappa}]$. The estimation cannot however be run having just one of the two parameters vary.

Technically, when only one parameter varies (or the two vary simultaneously in opposite directions), the algorithm cannot find an optimum. Intuitively, the problem is the link between ψ, ψ^* and $\bar{\kappa}$. Since the latter is imposed by calibration, having only one between ψ and ψ^* vary means that the remaining one is fixed by construction. In other words, which one is the parameter to identify is imposed by assumption.

To avoid making this assumption, I let both ψ and ψ^* vary. For this purpose, I define an equal (and large) grid for both parameters, I start with a dimension of the grid that satisfies $\psi + \psi^* < \bar{\kappa}$ and progressively enlarge the grid till $\psi + \psi^* = \bar{\kappa}$ (trying various combinations) and beyond. In such a way, I find an interior and significant optimum, as shown in Figure 1. The shape of the criterion function confirms that ψ is the only parameter that can be identified. The drawback of such large grid search is that the computational time needed to run the estimation increases substantially.

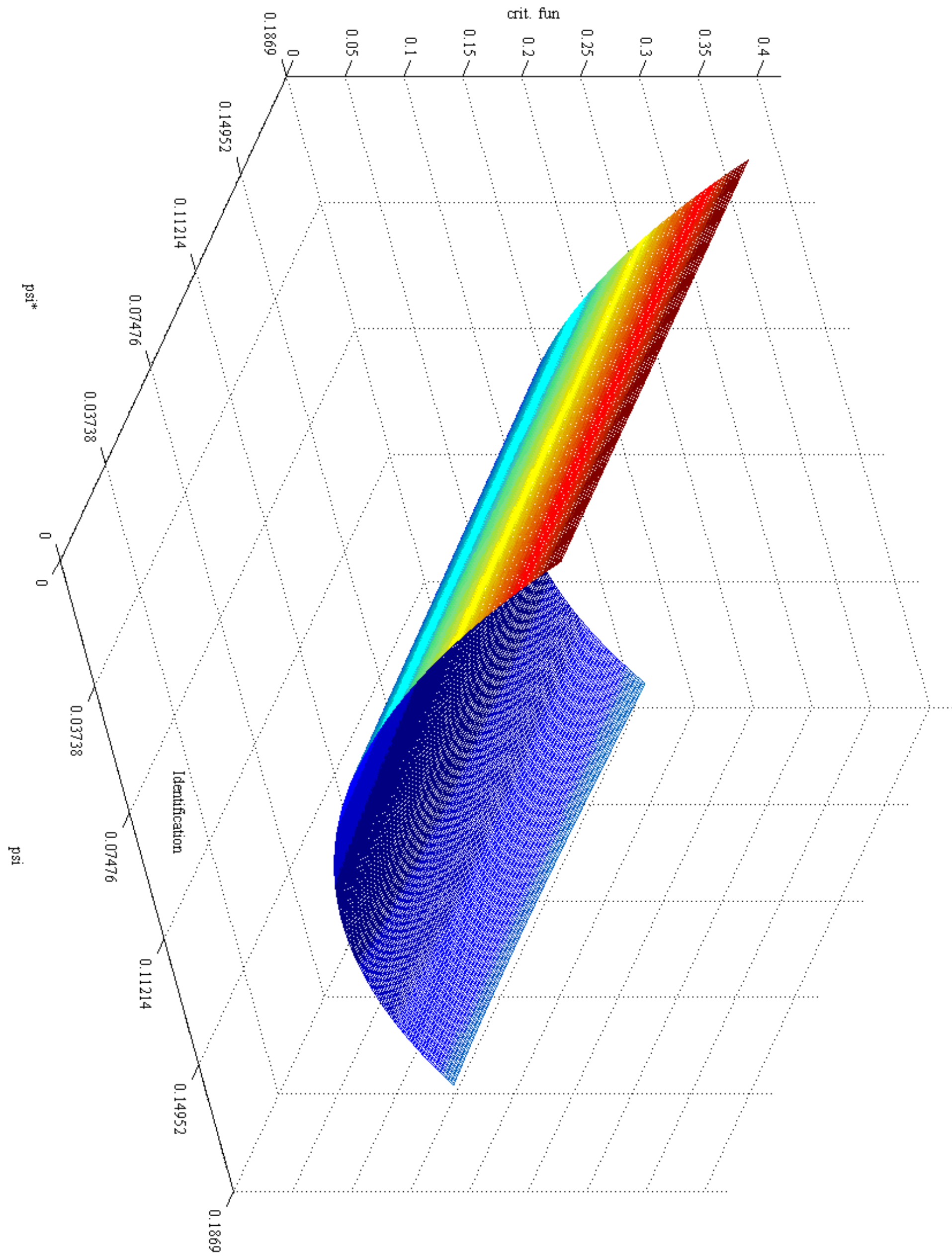

Figure 1: Determination of Coefficient(s) with the SMM Algorithm (N=30)

4 General Model Dynamics

The dynamics of the (linearized) model in section 2 depend on both the equilibrium portfolios and the corresponding estimate for ψ. The resulting behaviour of variables is best captured observing their reactions to a unitary (negative) impulse in home country productivity. Figure 2. is a general graph, that is, it refers to the full model featuring cross-border diversification of bond portfolios constrained by collateral as well as debt-to-asset ratios dependent on equity prices.

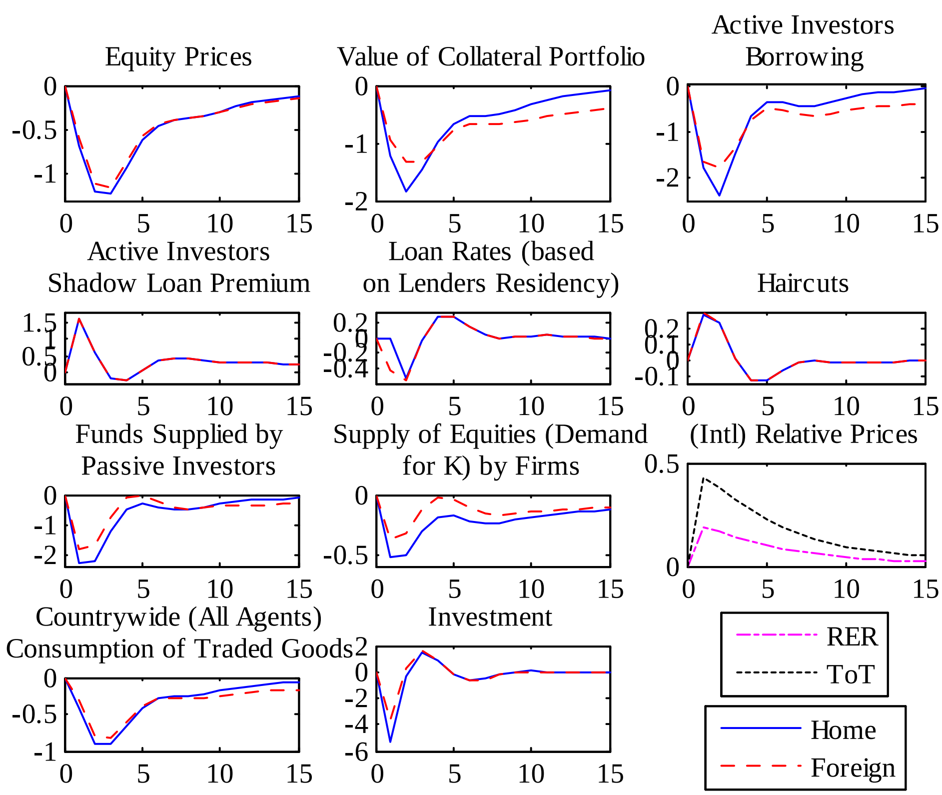

Figure 2: Responses to (Unitary) Shocks to Home Country Productivity

References

[1] Brunnermeier, M. and L. H. Pedersen (2009) "Market liquidity and funding liquidity", *Review of Financial Studies*, 22(6), 2201-2238.

[2] Canova, F. (2007) *Methods for Applied Macroeconomic Research*, Princeton University Press.

[3] Devereux, M. B. and A. Sutherland (2007) "Solving for Country Portfolios in Open Economy Macro Models", *HKIMR Working Paper* No.16.

[4] Devereux, M. B. and A. Sutherland (2011) "Country Portfolios in Open Economy Macro Models", *Journal of European Economic Association*, 9(2), 337-369.

[5] Duffie, D. and K. J. Singleton (1993) "Simulated Moments Estimation of Markov Models of Asset Prices", *Econometrica* 61(4), 929-952.

[6] Iacoviello, M. (2005) "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle", *American Economic Review*, 95(3), 739-764.

[7] Michaelides, A. and S. Ng (2000) "Estimating the Rational Expectations Model of Speculative Storage: a Monte Carlo Comparison of Three Simulation Estimators", *Journal of Econometrics*, 96(2), 231-266.

[8] Ruge-Murcia, F. (2007) "Methods to Estimate Dynamic Stochastic General Equilirbium Models", *Journal of Economic Dynamics and Control*, 31, 2599-2636.

[9] Tsay, R. (2005) *Analysis of Financial Time Series*, John Wiley and Sons.