Flow and Jamming of Granular Matter through an Orifice

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Abstract. When particles pass through an orifice, jamming can occur. The question is whether a jammed structure can be considered a new state of matter, and if the flow behaves differently when approaching jamming. An experiment, consisting of a silo filled with grains and an orifice at the base, is presented here. The jamming probability is measured, and it is shown that above a certain orifice size no jamming can occur. A power law divergence is found when that value is approached. Besides, the flow rate is different for small and large orifices. For large orifices, the Beverloo equation states that the flow depends on the diameter to the 5/2 power. But this relation breaks down for small orifices. A new functional dependence is proposed, in agreement with the experiments and the numerical simulations. Furthermore, the statistical analysis of the fluctuations for small orifices shows anomalous behavior.

1 Introduction

Although we all have an intuitive idea of jamming, arising maybe from our everyday experience with fragmented solids, it is not easy to provide a rigorous definition of it. Despite the fact that jamming is in some aspects similar to a liquid-solid phase transition, one of the key concepts involved is the fact that the constituent particles are kept in place by external mechanical stresses, instead of internal bonds between them. The mechanically stabilized structures involve long chains of grains, some of them forming arches that are sustained by the external forces [1]. When these forces are changed or cease to act, the structures loose their stability. The formulation of this idea has led to the definition of "fragile matter" [2], which in many cases is suitable for the physical systems that can get jammed.

Jamming has been recognized as a distinctive feature of granular matter [3]. A granular material (an assembly of a large number of solid particles, or grains, among which only contact forces are relevant) is just a paradigm of a system in which a mass flow can be arrested due to a jamming event. Glass-forming liquids, colloids and foams, among others, share many features with granular matter when their constituent elements get fixed and jam, forming an amorphous phase [4,5]. Traffic and pedestrian flow, insofar as it can be considered a current of solid particles, would also belong in this category. This opens interesting conceptual avenues to explore, such as the question of whether jamming can be defined as a phase transition, resulting in a new state of matter. The wealth of knowledge amassed about phase transitions could then be used to predict, control, avoid, undo or just describe jamming.

We will focus on a particular granular flow that can get jammed, namely, the outpouring of grains, driven by gravity, through an orifice at the base of a silo. By choosing this system, we are restricting ourselves to a particular case where some variables such as the density –which is of utmost relevance in many situations– are kept constant. But at the same time this allows us to explore methodically the features of the flow by changing just one parameter, which is the size of the outlet orifice.

The experiments begin with the grains pouring freely from the aperture at the base of the silo. Eventually, if the opening is not big (we will later address the meaning of 'big' in this context) the flow will stop due to the formation of an arch spanning over the opening. In the following we will call this an *avalanche*. The most readily available measurement is the size of the avalanche (the number of grains fallen) and its duration. The data obtained from a large number of avalanches allows us to describe the statistics of the jamming events. From the size of the avalanche and its duration, the mean flow can also be obtained. We will also be concerned with the dependence of the flow rate on the size of the outlet orifice.

Three questions will be addressed in this paper. The first one is whether there are or not two different regimes (i.e. a regime in which the flow will be arrested by arches of grains, and another regime in which the outpouring of grains will never stop), and if a well defined transition separates them. We will also look into the law for the mass flow rate. In fact, for large orifices an equation can be obtained that gives the number of beads fallen per unit time as a function of the size of the orifice. It is reasonable to ask if this relation is valid for small orifices, when jamming can occur, or if it fails, indicating that the features of the flow are different in this case. And finally, we will investigate if the grains move unusually when they are going to get jammed, by paying attention to their fluctuations. This knowledge could eventually be used to devise strategies to avoid jamming. Before we proceed, we will briefly describe our experimental devices, and we will gather some conclusions at the end of the paper.

2 Experimental Set-up and Numerical Procedures

The basic assembly we use in our experiments is an scaled cylindrical silo with a circular orifice at the base. Beneath it, a scales is placed to measure the weight of the grains. The number of grains in an avalanche can then be calculated. We also measure its duration by means of a microphone that detects the sound of the falling grains.

The filling procedure of the silo is always the same, so as to begin with the same initial conditions in every run. The avalanche is started by means of an air jet aimed from below at the exit orifice. The air jet does not disturb the bulk of the matter contained in the silo, and only a local perturbation is produced that breaks the arch blocking the orifice. Moreover, due to the Janssen effect [6], the pressure at the bottom of the silo does not change much –provided that the height reached by the grains is more than twice the diameter of the silo. We have also checked that the silo is big enough so that the lateral walls do not influence the results.

The experimental set-up has previously been described, so we refer the interested reader to a previous paper [7] for additional details. A sketch is provided in Fig. 1.

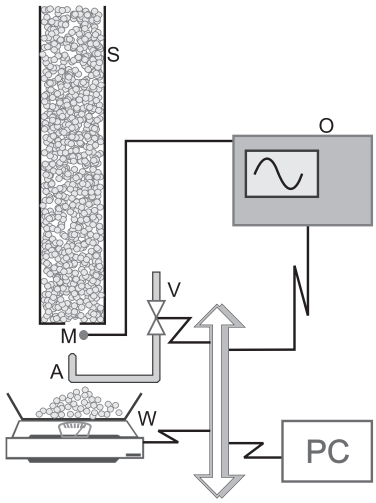

Fig. 1. Sketch of the experimental setup for the cylindrical silo. S: silo; M: microphone; A: compressed air; O: oscilloscope; V: valve; W: scales

We have also used a two-dimensional silo to conduct experiments. This container consists of two glass panes between which a gap slightly larger than the bead diameter is left. Again, a scales beneath the opening at the base is used to measure the size of the avalanche. Its duration is obtained in this case through a photodetector that registers the time that a light beam has been blocked by falling grains. The experimental procedures are therefore much similar to those in the cylindrical silo. We have used a high-speed camera (Photron model Fastcam 1024, capable of recording a window of 512x512 pixels at 3000 frames per second) to track the beads inside this silo. In this way, we have obtained the velocity profile of the grains inside the silo, we have studied the fluctuations of grains as they move towards the outlet and we have also measured the instantaneous flow rate. This device will be described in more detail in a forthcoming article [8].

In order to keep the experiment as simple as possible, we have used smooth monodisperse spherical beads most of the time (see Table 1). As the beads are big enough, we can safely ignore cohesive forces (arising from humidity, electrostatic charge, etc.). In some runs, we have used other kinds of beads in order to test the robustness of our results: polydisperse beads, rough spheres, and irregular granular materials, such as sand, rice, lentils and others. It will be indicated when needed. Most of the results presented in the following do not depend on the bead material or size (otherwise it will be specifically stated). In fact, the relevant parameter is $R = \frac{D}{\phi}$, the ratio between the diameter of the outlet orifice D and the diameter of the beads ϕ. We have checked this in a variety of situations [9].

Table 1. Beads used in the experiments

Material	Nominal diameter (mm)	Diameter (mm)	Density (g/cm^3)
glass	0.5	0.42 ± 0.05	2.2 ± 0.1
glass	1	1.04 ± 0.01	2.4 ± 0.1
glass	2	2.06 ± 0.02	2.2 ± 0.1
glass	3	3.04 ± 0.02	2.4 ± 0.1
lead	2	1.98 ± 0.06	11.4 ± 0.5
lead	3	3.0 ± 0.1	10.9 ± 0.5
Delrin	3	3.00 ± 0.02	1.34 ± 0.05
Stainless steel	1	1.00 ± 0.01	7.6 ± 0.3

We have also carried out computer simulations of disks in two dimensions using soft particle molecular dynamics [10]. The details of the repulsive forces and the dissipative terms used in the model can be found in [11]. The numerical experiment involves about 5000 disks (compared with more than two hundred thousands in real experiments), but it allows us to enlarge the range of parameters in which we can obtain data, and also the resolution is much bigger.

3 Jamming Probability

The data from which we start the examination of the jamming probability is the statistics of avalanches. For each adimensional size of the orifice R we recorded typically about 3000 avalanches, although in a few cases ten times more were registered. The histogram for the number of beads in the avalanche (its size s) is shown in Fig. 2, where the number of occurrences of the avalanche of size s for an orifice of size R is called $n_R(s)$. The histograms for other orifice sizes are similar; the only conspicuous difference are the number of very small avalanches (i.e. the portion of the histogram to the left of the maximum). If we ignore these few points (which are not relevant unless the outlet orifice is of a size comparable to the bead diameter), then the histogram shows an exponential tail, as evidenced in the inset of Fig. 2. This means that the phenomenon under study is governed by a characteristic magnitude (such as, for instance, the mean avalanche size, the mean avalanche duration, or others, as long as they are related).

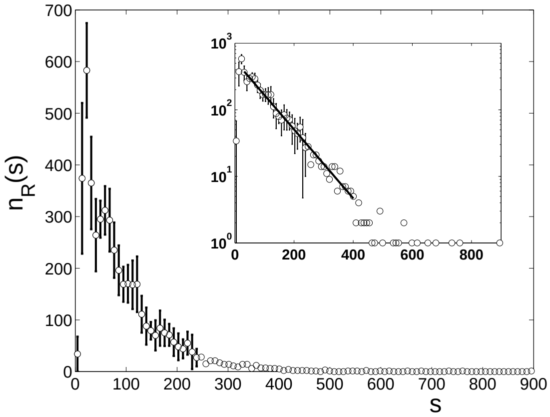

Fig. 2. Histogram of the avalanche size s (number of beads fallen) for an orifice $R = 3$. The inset shows the same data in semilogarithmic scale with a linear fit (*solid line*)

The existence of a characteristic parameter allows us to rescale the histograms, using for example the mean avalanche size $\langle s \rangle$. As expected, all the histograms collapse now in a single curve (see Fig. 3), except for the very small avalanches, as noted before.

A simple model to describe the statistics of avalanches can be built from these premises [12], and it will lead to a simple mathematical expression of the jamming probability. Let us start by assuming that the probability of a bead to get jammed when passing through the outlet orifice does not depend on the probability that nearby beads get jammed. This amounts to consider that these events are independent. In this model, we can represent the outflow as a linear series of events, each one corresponding to a bead that either passes through the orifice without getting jammed, with probability p, or it gets jammed at the orifice with probability $1 - p$ (and the flow stops). Then, the probability of finding an avalanche of size s for a given R –which we have called $n_R(s)$ above– is equal to the probability of s beads falling through the orifice followed by a bead that gets jammed, i.e.

$$n_R(s) = p^s \; (1 - p) \; \Rightarrow \; \log\left(n_R(s)\right) = s \log(p) + \log(1 - p) \qquad (1)$$

This expression allows us to obtain p from the linear fit of the histogram as displayed in the inset of Fig. 2: the slope of the straight line is just $\log(p)$. Note that p is also related to the mean avalanche size, because p is the total number

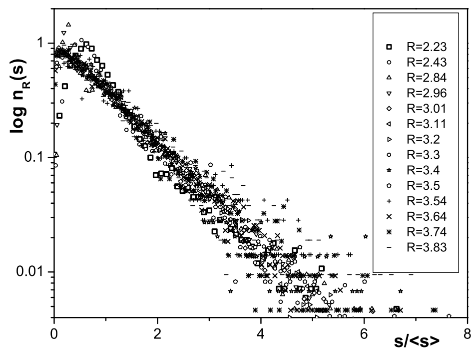

Fig. 3. Normalized histogram for a series of openings R as indicated in the legend, in semilogarithmic scale. The rescaling has been performed dividing in each histogram the avalanche size s by the mean avalanche $\langle s \rangle$ corresponding to that particular R

of beads fallen in a series of avalanches divided by the number of fallen beads plus the number of jamming events, and then $p = \langle s \rangle / (\langle s \rangle + 1)$.

Let us now define the *jamming probability* J as the probability that the flow gets arrested before N beads fall. Obviously, the jamming probability depends on N and on the size of the outlet orifice R. We can write

$$J_N(R) = 1 - \sum_{s=N}^{\infty} n_R(s) \tag{2}$$

which is just the statement leading to the definition of J, i.e. the probability that the avalanche is smaller than N.

Let us now rescale the avalanche size by $\langle s \rangle$, so that

$$s^* = s / \langle s \rangle \tag{3}$$

$$n_R^*(s^*) = \langle s \rangle \, n_R(s) \tag{4}$$

If $\langle s \rangle \gg 1$, substituting p in terms of $\langle s \rangle$ we get

$$n_R^*(s^*) = \left(1 + \langle s \rangle^{-1}\right)^{-1} \exp\left[-s^* \langle s \rangle \ln\left(1 + \langle s \rangle^{-1}\right)\right] \rightarrow e^{-s^*} \tag{5}$$

and

$$J_N(R) = 1 - p^N = 1 - \exp\left[-N \ln\left(1 + \langle s \rangle^{-1}\right)\right] \rightarrow 1 - e^{-N/\langle s \rangle} \tag{6}$$

Equation (5) is just the expression of an exponential tail as presented in Fig. 3. We will now turn to (6) and see how it compares to the experimental values obtained for J. In order to do this, we have collected data for about fifty different R. If we fix N at some particular value, we can calculate from the data J_N for each R. For instance, in Fig 4**(a)** we plot J for $N = 100$ beads in a range of outlet orifices going from about $R = 1.3$ to $R = 4.3$. If we look at small R, say $R = 2$, the value of J_{100} is almost 1, meaning that for such a small orifice the probability that it jams before 100 beads have fallen is very high. For large orifices (say, $R = 4$) J is almost 0, so that there is a small probability that the outpouring of beads will get arrested before 100 beads fall. At about $R \approx 3$ the jamming probability is 1/2. The solid line is the fit provided by (6). Note that there are no free parameters in the fit, as $\langle s \rangle$ is also obtained experimentally.

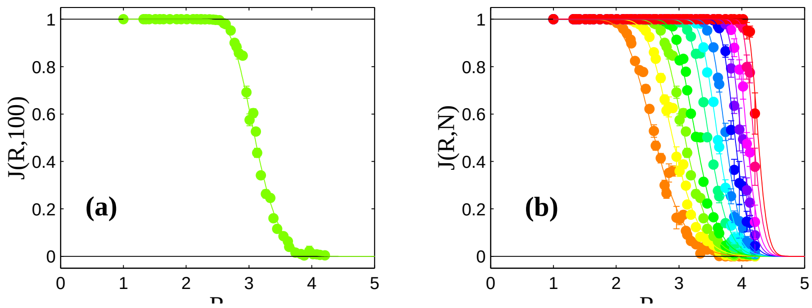

Fig. 4. The jamming probability J as a function of R. (**a**) $J(R)$ for $N = 100$. (**b**) $J(R)$ for $N = 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000, 20000, 50000, 100000$ *(color changes from orange to red as* N *increases)*. The solid lines are the fits given by (6)

The same can be done for different values of N, which are displayed with the corresponding fits given by (6) in Fig 4**(b)**. Let us remark that the agreement between the model and the experimental data does not validate the model; it merely indicates that the model can be used as a good approximation. There are indeed other more complicated models, with some theoretical foundations, that also agree with the experimental data [13].

One interesting feature that can readily be observed in Fig 4**(b)** is that J tends to a step function as N increases. Note that the series of N used in the plot is not a linear progression, so this tendency is really marked. Let us state the same idea in a different way. For a very large number of beads ($N \to \infty$), there are only two possibilities: either the orifice gets blocked, with probability very close to one, for small R, or the flow will never jam, even after waiting for very long times, if the size of the orifice is big.

The value of R separating the two situations is called the *critical radius* R_c because it is found that the mean avalanche size diverges as a power law as this value is approached [9]. Let us remark that although this divergency is found in some phase transitions, the inverse is not true, i.e. its existence does not warrant to conclude the presence of a phase transition. It is nevertheless enough to show the existence of two different regimes, one of them where the flow will eventually stop, and another in which it will never jam.

The value of R_c can be calculated from the mean avalanche size:

$$\langle s \rangle = \frac{A}{(R - R_c)^\gamma} \tag{7}$$

where A, R_c and γ are free fitting parameters. Both of them depend on the shape of the grains, but not on other characteristics such as density or roughness. For spherical grains, it has been obtained [7] that $R_c = 4.94 \pm 0.03$.

Even though A, R_c and γ depend on the shape of the grains, the power law is always observed (see Fig. 5), so even in these cases the existence of a critical radius can be asserted. The value of γ is usually much higher than in "typical" phase transitions; for instance, it is $\gamma = 6.9 \pm 0.2$ for glass spheres.

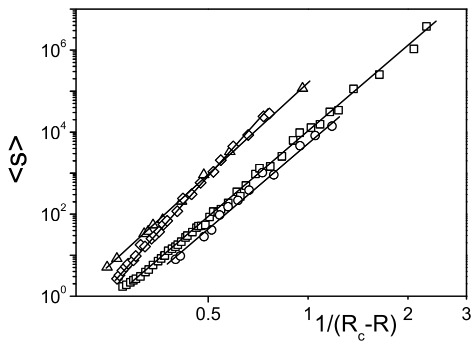

Fig. 5. The dependency of the mean avalanche size $\langle s \rangle$ on R for rice (*triangles*), lentils (*diamonds*), spherical glass beads (*squares*) and irregular pasta grains (*circles*)

4 The Flow Rate at the Outlet

Let us now turn to the question of the mass flow rate at the outlet of the orifice. In particular, we investigate whether the flow rate law changes when the exit orifice is small ('small' meaning $R \simeq R_c$, as explained in the previous section).

The most widely accepted expression for the mass rate of particles is known as Beverloo's law [14,15]. This equation is valid for the outpouring of grains through an orifice due to gravity, and it can be obtained from a dimensional analysis of the problem. Earlier experiments had shown that if the dimensions of the silo, the size of the beads and the diameter of the outlet orifice fulfilled certain geometrical conditions, then the flow rate is independent on the details of the container. Let us then suppose that the mass flow rate W depends on the density of the granular material ϱ, the acceleration due to gravity g, the orifice diameter D and the friction coefficient μ. A simple calculation leads to the conclusion that the only permissible relationship is the following:

$$W = C(\mu)\ \varrho\ \sqrt{g}\ D^{5/2} \tag{8}$$

There is another line of reasoning leading to the same formula. The flow rate must be proportional to the product of the velocity v times the area of the orifice. If it is assumed that the grains fall freely from a height $D/2$, then $W \propto v\ D^2$ and therefore $W \propto \sqrt{g}\, D^{5/2}$. Note that in a two dimensional silo with a slit at the base, $W \propto D^{3/2}$

However, the scaling law $D^{5/2}$ is at odds with experimental results. In an attempt to rescue it, the concept of "empty annulus" [16] was taken in the formula. It means that the grains in fact do not use the whole orifice, but an effective exit aperture given by $D - k\phi$, where k is a free parameter. Using values of k between 1 and 3, many experimental results have been fitted with this equation:

$$W = C(\mu)\ \varrho\ \sqrt{g}\ \left(D - k\phi\right)^{5/2} \tag{9}$$

which is generally known as the Beverloo law.

This formula, however, has only been checked for a small range of outlet sizes, and for big values of R. We present for the first time the mass flow rate values for a large range of R. In order to combine in a single graph data from different materials, we divide the flow by the mass of one bead, so W_b is now the flow rate in number of beads per unit time. Our data are shown in Fig. 6.

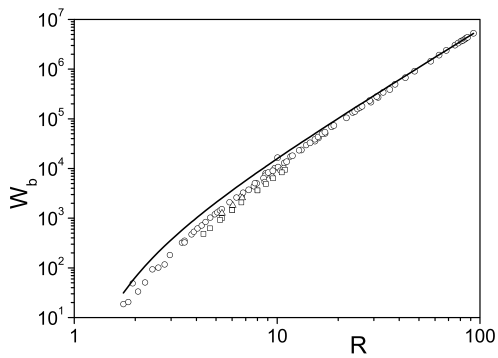

Fig. 6. The flow rate as a function of R for different spherical grains, in logarithmic scale: glass (*circles*), lead (*triangles*) and Delrin (*squares*). The solid line is a fit with (9)

As expected, there is a small dependency of W_b on the material, probably due to the friction coefficient, but surely not to ϱ (Delrin is lighter than glass while lead is heavier, and for both W_b is smaller than for glass).

It turns out that it is not possible to fit these data with Beverloo's equation (9). The only possible way to retain the scaling $W \propto R^{5/2}$ in agreement with the data for big R is to take $k = 1$ [8], which is the fit shown in Fig. 6. But then the flow rate predicted by (9) for small R depart noticeably from the experimental results.

The reason why Beverloo's formula (9) enjoys such a wide acceptance is because it can fit the data if only a small enough range of R is considered. This is more clearly displayed if we plot $W_b^{2/5}$ vs. R (Fig. 7). The intercept of a linear fit on the axis gives the value of k. In Fig. 7 we show such a fit for the data corresponding to $R > 50$, which is clearly unsatisfactory for small R. Any small interval of R can be reasonably fit with a straight line, and the fitting parameters provide the values for C and k. If a large interval is considered, however, a linear fit is not acceptable.

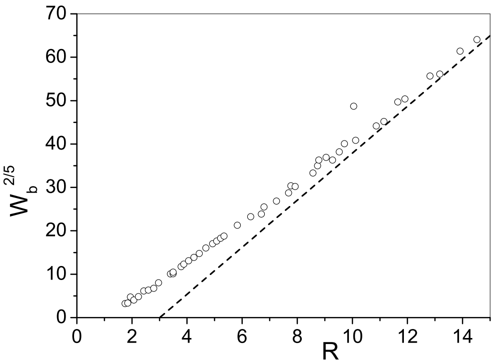

Fig. 7. $W_b^{2/5}$ as a function of R for small outlet sizes (only data for glass beads is shown). The dashed line is a linear fit taking only the data for $R > 50$

It is clear that (9) must be modified to agree with the experimental results. We have calculated the deviation of the prediction given by this formula with respect to the experimental result, and we have shown that it has the form of a negative exponential [8]. We therefore propose a new formula for the flow rate:

$$W_b = C' \left(1 - \frac{1}{2} e^{-b\cdot(R-1)}\right) (R-1)^{5/2} \tag{10}$$

The fit obtained with this formula is shown in Fig. 8.

In summary, we have shown that the parameter k, which was introduced arbitrarily in the equation for the mass flow rate, is not a valid option in the sense that it does not give an acceptable result for a large range of orifice sizes. We propose instead a new formula which fits nicely the experimental data over

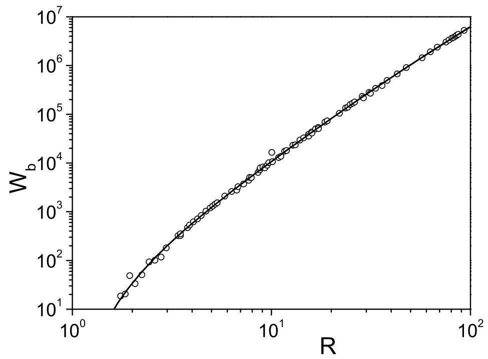

Fig. 8. The data for glass beads (same as in Fig. 6) fitted with (10). The values of the constants are $C' = 64$ and $b = 0.05$

almost two decades of R. Note that $k = 1$ makes sense as W_b must tend to zero as $R \to 1$. This correction is only relevant for small values of R, and the asymptotic scaling $R^{5/2}$ is recovered for large values of R.

5 The Movement of Particles inside the Silo

Let us now address the issue of describing the movement of the grains inside the silo. Several models exist for the velocity field inside the silo, and most end with the same expression for the mean vertical velocity (although the hypothesis and assumptions are often quite different). Let us follow the line of reasoning proposed by Nedderman and Tüzün [17] to derive their "kinematic model". The simplest relation that makes sense between the horizontal and vertical components of the velocity (u and v, respectively) is

$$u = -B\,\frac{\partial v}{\partial x} \tag{11}$$

which means that the particles tend to be dragged to those zones where the vertical velocity is higher. Justifications to this formula have been offered in terms of the Reynolds dilatancy principle or from statistical arguments [18,19], but its theoretical foundation remains precarious. Remark that therefore the meaning of the parameter B is unclear.

Coupling this with the assumption that the continuity equation holds for granular media, an expression for the vertical velocity is obtained:

$$v = -\frac{Q}{\sqrt{4\pi B y}} \exp\left(-\frac{x^2}{4By}\right) \tag{12}$$

where Q is the flow rate through the exit orifice, and x and y are respectively the horizontal and vertical coordinates, taking the center of the orifice as the origin.

We have measured this profile by tracking the grains inside the silo. As we need visual access to the particles, we turned to the two dimensional silo described above, in which the beads can be tracked with high resolution. In order to avoid temporal or spatial short-range correlations, which are prone to appear in two dimensional silos, we have performed 24 runs, every one involving the tracking of at least 3000 particles, to obtain each velocity profile. Our results are shown in Fig. 9, which are the vertical velocities across the silo at four different heights measured for an orifice size $R = 15.8$.

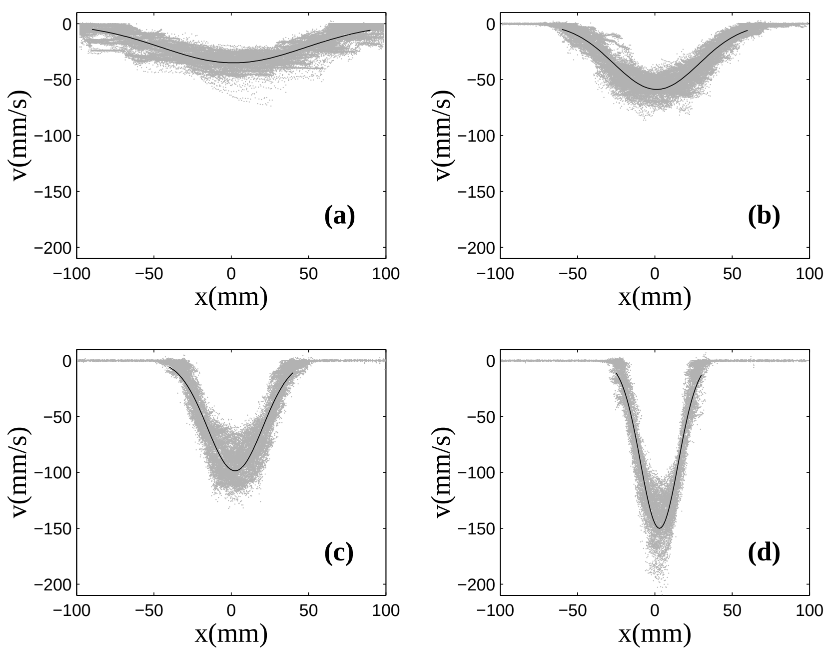

Fig. 9. Vertical velocity profiles inside the silo at different heights: **(a)** $y = 355$ mm; **(b)** $y = 155$ mm; **(c)** $y = 65$ mm; **(d)** $y = 30$ mm. All the plots are at the same scale. Solid lines are fits using (12). The diameter of the orifice is $D = 15.8$ mm and the diameter of the beads is $\phi = 1$ mm.

It can be seen that the formula (12) reproduces quite well the experimental results. This is only true, however, for the mean velocity profile: there is a large dispersion of the velocity around its mean value. Further agreement comes from the fact that the variable Q (which was a free parameter of the fit) and the measured flow rate at the outlet differ by 10% at most.

It is interesting to study the behavior of the parameter B (sometimes called the kinematic parameter) as a function of the height above the orifice (see Fig. 10). This relationship had previously been reported, and while some authors propose an exponential growth [20] compatible with our data, others have found otherwise [21,22].

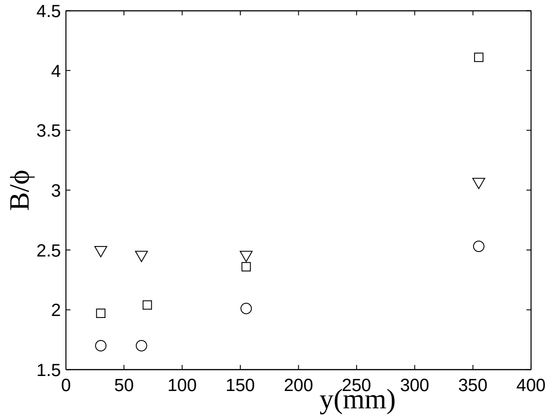

Fig. 10. The value of the parameter B (divided by the bead diameter to make it dimensionless) as a function of the height for three different orifice sizes: $R = 4.8$ (*squares*), $R = 9.5$ (*circles*) and $R = 15.8$ (*triangles*)

There have been several attempts to provide a physical meaning for the parameter B. By assuming that there are characteristic time and length scales in the movement of the particles, B can be identified with a diffusive length [18,19], but the value found is way off the experimental result. In order to correct this disagreement, a model for the diffusion of voids inside the granular material (where the void is "shared" among several grains that move collectively, an ensemble called a "spot") has been proposed [23]. By adjusting the size of the spot, a value of B compatible with experiments can be found. Nevertheless, there is no sound reason for assigning a size to the spot other than recovering the experimental result. This issue remains still unclear.

We have paid special attention to the fact that there is a large dispersion of velocities around its mean local value. All the above cited models in some way or another lead to the prediction of gaussian fluctuations and normal diffusion for the particles as they move downwards inside the silo. Some recent results, however, do not agree with this picture [24].

We have performed numerical simulations (as explained in Section 2) using discrete element modeling [25,11]. One of the most interesting features of the motion of the particles is that there is a transition from a ballistic behavior at small time scales to a diffusive behavior at large time scales. (The time is rescaled by normalizing it to the time it takes for a bead to fall its own diameter, i.e.

multiplying it by the factor v_y/ϕ). This happens at least for large R (and it is not clear at the moment if the same can be said for small R). This result is displayed in Fig. 11. The numerical simulations agree with the experiment.

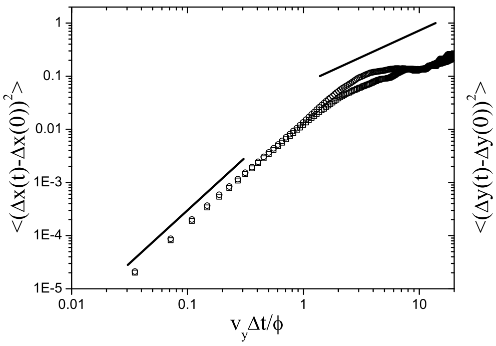

Fig. 11. Mean square displacements for $R = 16$ as a function of time (note that time has been rescaled, so that one corresponds to the time it takes for a bead to fall a distance equal to its own diameter). The mean square displacements for the vertical direction (*squares*) and for the horizontal direction (*circles*) both show a ballistic behavior at short time scales and a diffusive behavior at long time scales (the solid lines have slopes 2 and 1)

We have also found, both experimentally and numerically, that the position fluctuations are not gaussian. This happens for both large and small R, although we do not know whether there are differences between both situations. We show an example of these fluctuations in Fig. 12.

It is clear that anomalous behavior (in the sense of non-gaussian fluctuations and diffusion departing from normal) is present. The extent to which this influences jamming is being actively studied.

6 Conclusions

In this paper we have addressed the issue of jamming in a granular flow, driven by gravity, when it finds an obstacle in the form of an orifice. In this simple system, some conclusions can be reached. As they are quite general, it would be interesting to check their validity in other situations beyond the one presented here. Let us recall that in our experiments we have kept constant the density of the granular material (i.e. the compaction fraction).

We have shown that the phenomenon is governed by a characteristic parameter. As there exist a relationship among several relevant variables, one of them can be chosen without lack of generality. We have singled out the mean

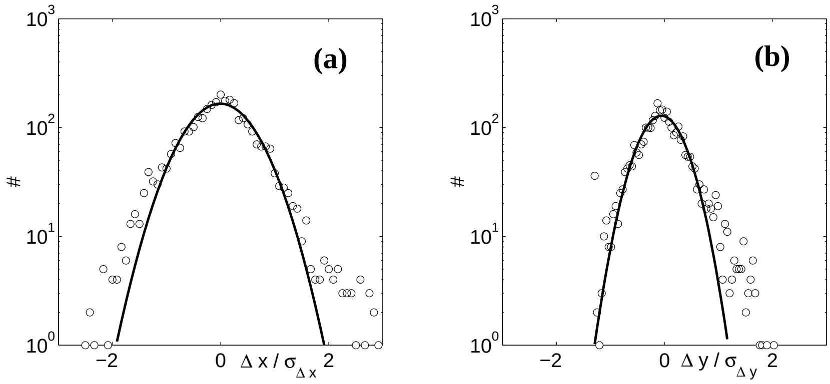

Fig. 12. The histogram for the fluctuations of the particle positions. They correspond to the fluctuations in the horizontal coordinate **(a)** and the vertical coordinate **(b)**, for $R = 15.9$. They are at the same scale and normalized by the standard deviation. The solid lines are the gaussian best fits

avalanche size $\langle s \rangle$. It has been found that $\langle s \rangle$ tends to infinity as a given value of the orifice size R_c is approached. We call this value *critical radius* because a power law divergency is found. Therefore, there are two distinct possibilities: either a jamming will appear eventually if the orifice size is smaller than R_c, or a jamming will never take place –even after waiting for a very long time– if $R > R_c$. This fact is universal and does not depend on the material properties of the grains (such as density, roughness, and so on), but the particular value of R_c does depend on the shape of the grains.

We have studied the behavior of the mass flow rate W through the exit orifice. We have shown that the scaling $W \to R^{5/2}$ is asymptotically valid for big orifices (big meaning $R \gg R_c$). Nevertheless, the correction usually introduced in Beverloo's equation (stemming from the notion of an effective aperture, and its reduction in the shape of an "empty annulus") is not valid in the sense that it cannot reproduce the experimental results spanning over a large range of R. We instead propose another formula that corrects the value of W for small orifices, by a multiplicative term involving an exponential. We cannot offer at the moment a meaning for the parameters in this correction term; this issue must be explored further.

The motion of individual particles inside the container also reveals interesting features. At long time scales, the shape of the mean velocity profile is correctly described by diffusive models. But the behavior of individual particles at short time scales shows non-gaussian fluctuations and anomalous diffusion, which is at odds with the mentioned models. We have shown that for small R there exist particular features of the flow rate, and jamming can appear, blocking the flow. We do not know if there are specific characteristics of the motion of particles at small time scales when the exit orifice is small; we are looking into this issue, because it can give hints on how to prevent the occurrence of jamming.

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References

1. L. A. Pugnaloni, G. C. Barker, A. Mehta: Advances in Complex Systems **4**, 289 (2001)
2. M. E. Cates, J. P. Wittmer, J.-P. Bouchaud, P. Claudin: Physica A **263**, 354 (1999)
3. A. Liu, S. Nagel: Nature **396**, 21 (1998)
4. E. R. Weeks, *Soft jammed materials*, submitted to Tohoku University Press. Available from http://www.physics.emory.edu/˜weeks/lab/pubs.html
5. A. Liu, S. Nagel (eds.): *Jamming and rheology* (Taylor and Francis, London 2001)
6. J. Duran: *Sables, poudres et grains* (Eyrolles, Paris 1997)
7. I. Zuriguel, A. Garcimartín, D. Maza, L. A. Pugnaloni, J. M. Pastor: Phys. Rev. E **71**, 051303 (2005)
8. C. Mankoc, A. Janda, R. Arévalo, J. M. Pastor, I. Zuriguel, A. Garcimartín, D. Maza: *submitted to Granular Matter*
9. I. Zuriguel, L. A. Pugnaloni, A. Garcimartín, D. Maza: Phys. Rev. E RC **68**, 030301 (2003)
10. J. Schäfer, S. Dippel, D. E. Wolf: J. Phys I(France), **6**, 5 (1996)
11. R. Arévalo, A. Garcimartín, D. Maza: Eur. Phys. J. E **23**, 191 (2007)
12. A. Santos, *private communication*
13. K. To: Phys. Rev. E **71**, 060301R (2005)
14. W. A. Beverloo, H. A. Leniger, J. van de Velde: Chem. Eng. Sci. **15**, 260 (1961)
15. R.M. Nedderman: *Statics and Kinematics of Granular Materials* (Cambridge University Press, Cambridge 1992)
16. R. L. Brown, J. C. Richards: Trans. Instn. Chem. Engrs. **38**, 243 (1960)
17. R. M. Nedderman, U. Tüzün: Powder Technol. **22**, 243 (1979)
18. J. Litwiniszyn: Bull. Acad. Pol. Sci. **11**, 593 (1963)
19. W. W. Mullins: J. Appl. Phys. **43**, 665 (1972)
20. A. Medina, J. A. Córdova, E. Luna, C. Treviño: Phys. Lett. A **250**, 111 (1998)
21. I. Sielamowicz, *private communication*
22. K. D. Kafui, C. Thornton: 'Some observations on granular flow in hoppers and silos'. In: *Powders & Grains 97, 3rd International Conference on Powders and Grains at Durham, North Carolina, U.S.A., 18-23 May, 1997)*, ed. by R. P. Behringer, J. T. Jenkins (Balkema, Rotterdam 1997) pp. 511–514
23. M. Z. Bazant: Mechanics of Materials **38**, 717 (2006)
24. J. Choi, A. Kudrolli, R. R. Rosales, M. Z. Bazant: Phys. Rev. Lett. **92**, 174301 (2004).
25. D. Maza, A. Garcimartín, R. Arévalo: Eur. Phys. J. Special Topics **143**, 191 (2007)