Editors' Suggestion

Decoupling Geometrical and Kinematic Contributions to the Silo Clogging Process

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Based on the implementation of a novel silo discharge procedure, we are able to control the grains velocities regardless of the outlet size. This allows isolating the geometrical and kinematic contributions to the clogging process. We find that, for a given outlet size, reducing the grains velocities to extremely low values leads to a clogging probability increment of almost two orders of magnitude, hence revealing the importance of particle kinematics in the silo clogging process. Then, we explore the contribution of both variables, outlet size and grains velocity, and we find that our results agree with an already known exponential expression that relates clogging probability with outlet size. We propose a modification of such expression revealing that only two parameters are necessary to fit all the data: one is related with the geometry of the problem, and the other with the grains kinematics.

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The passage of many-particle systems through confined geometries like porous substrates [1,2], bottlenecks [3], or pipes [4] can be interrupted by the formation of clogs, local structures that trigger the complete arrest of the flow. Clogging, a phenomenon that resembles jamming but presents intriguing differences [5–7], has been recently studied for the simplest case of a single orifice in scenarios of a diverse nature, such as microbial populations [8], microparticles [9,10], droplets [11], crowds of pedestrians [12], or sheep flocks [13,14]. Despite all these works, the mechanisms governing the clogging process are unclear.

Presently, it is accepted that the probability of building up a stable arch does not depend on time, hence leading to a exponential distribution of flowing intervals or avalanche sizes. Also, it is well known that the clogging probability dramatically reduces when increasing the outlet size, yet there is not consensus about the existence of a critical outlet size above which the flow may never get interrupted [15–18]. Indeed, in the last years, a nondivergent expression where the average avalanche size increases exponentially with the outlet size raised to the dimensionality of the problem is gaining supporters [10,16–18]. Beyond this discussion, some studies have recently shed light on the way in which clogging is affected when varying several parameters such as particle softness and friction [11,19], particle shape [20,21], width of the silo [22], outlet shape [23,24], presence of obstacles [25,26], or interstitial fluid [27].

Surprisingly, all the studies on silo clogging implemented so far, have dealt with the case of grains purely discharged by gravity. And this is so despite that in industrial silos (integrated in production lines) the extraction of grains is commonly performed by means of a conveyor belt which is able to regulate the outflow of particles. Apart from its applied interest, this type of silo discharge is of fundamental importance because it allows controlling the particles velocities independently on the outlet size (in a gravity discharged silo the beads velocities scale with $\sqrt{gD}$, where $D = 2R$ is the orifice diameter and g is gravity acceleration [28]). This velocity control is also interesting in order to establish analogies with most of the systems mentioned above that are precisely characterized by a constant velocity of the agents [8–13]. A similar velocity controlled flow through bottlenecks was implemented in [29] to demonstrate that the pressure at the bottom of a silo does not determine the flow rate. Going back to the clogging problem, it should be noted that the particles velocities have been suggested to be behind the effect of, for example, silo width [22] and interstitial fluid [27]. Nevertheless, the only works in which this parameter has been investigated have done it indirectly: through a change in the system effective gravity that led to a very limited impact on clogging [28,30,31].

The experimental setup is a quasi two-dimensional silo as the one used in Ref. [32] in which a conveyor belt is placed below the orifice (Fig. 1). The silo has been made of two transparent glass sheets separated by two aluminum blanks of 4 mm supplemented by thin pieces of cardboard. These guarantee that the particles, monodisperse stainless steel spheres of diameter $d_p = 4$ mm, arrange in a single layer. The blanks play also the role of silo lateral walls leading to a silo dimensions of 61.2×160 cm^2. At the bottom of the silo there are two movable wedge-shaped pieces made of stainless steel whose separation defines the orifice size D. Below the orifice, a conveyor belt made of honeycomb-shaped rubber is able to provide an extraction velocity v_{belt} ranged between 0.1 and 16 cm/s (Fig. 1). The location of the belt has been fixed at a distance of $h = 3.2 \pm 0.1$ mm between the honeycomb upper protrusions and the lower edges of the glass sheets (the role of this

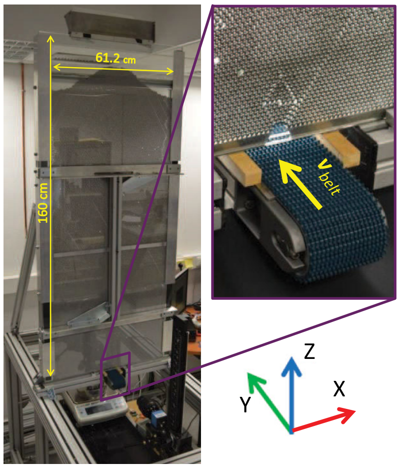

FIG. 1. Experimental setup. The close up shows the orifice region with the conveyor belt placed below. The yellow arrow indicates the belt direction of movement. The origin of coordinates system is set at the center of the orifice.

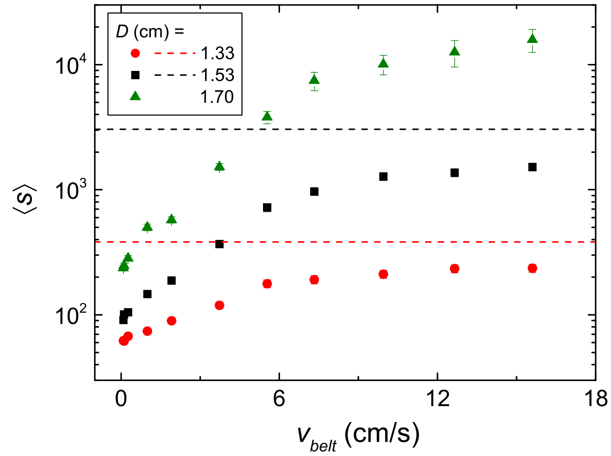

FIG. 2. Experimental data of the mean avalanche size as function of the belt velocity for the three outlet sizes indicated in the legend. In all cases we use spheres of diameter $d_p = 4$ mm and a gap between the belt and the silo of $h = 3.2 \pm 0.1$ mm. The horizontal lines correspond to the case where the silos are discharged only by gravity (without conveyor belt). Note the logarithmic scale in the y axis.

parameter is discussed in [33]). At the end of the belt the material falls into a box placed on top of a balance. A camera in front of the orifice region is used to detect clogging arches, and a vibrator hitting the glass sheet is implemented to destroy them.

The experiment starts when we switch on the conveyor belt and the grains begin flowing out the silo. Once a clog is formed, the camera detects it and registers the avalanche duration. Then, the belt is left running until all the grains fall into the box and the avalanche mass is registered. From this, the avalanche size in number of beads, s, is obtained. Finally, the vibrator breaks the arch and the belt starts moving, repeating the process as many times as required (around 1000 for each experimental condition). When the grains level within the silo falls below two times the silo width, the silo is filled through a hopper placed at the top. Moreover, in order to relate the clogging magnitudes to grains velocities, we have taken videos of the outlet region during the flowing intervals. From these, we have obtained the centroids and velocities of all beads as well as the mean flow rate through the exit line. The frame rates implemented range between 125 and 500 fps, a frequency that has been checked to be high enough to capture the dynamics of the particles for each experimental scenario [33]. Finally, we have also performed some control tests for the case where the grains flow out of the silo freely under the action of gravity (without extracting belt).

Figure 2 shows the dependence of the mean avalanche size $\langle s \rangle$ on the belt velocity. The influence of this parameter on clogging is evident for all the orifices and it becomes more significant as the outlet size increases. Indeed, for the largest outlet investigated, the variation of $\langle s \rangle$ between the lowest and highest belt velocities is close to two orders of magnitude. It is also interesting to note that $\langle s \rangle$ tends to nonzero values when v_{belt} approaches zero in a similar way than in [30,31].

Aiming at shedding light on the physical origin of the effect of the belt velocity on the clogging development, we first analyze the way in which the belt velocity affects the flow and particle dynamics. In Fig. 3(a) we report the values of flow rate W versus the belt velocity for the three outlet sizes investigated. For low belt velocities, the flow rate increases linearly with v_{belt} starting from $W = 0$ when $v_{\mathrm{belt}} = 0$. Then, for sufficiently large belt velocities, the flow rate seems to saturate at values that depend on the orifice size and are always below the flow rate that would correspond to the free discharge case. In order to check the effect of the belt velocity on the particles' motion, the mean vertical velocity of the grains $\langle \mathbf{v}_z \rangle$ is represented versus v_{belt} [Fig. 3(b)]. This magnitude has been calculated as the arithmetic mean of the velocity of all particles crossing a horizontal $D \times 0.5 d_p$ window centered at $x = 0$. The outcomes are similar to the ones corresponding to the flow rate: $\langle \mathbf{v}_z \rangle$ increases near linearly for low belt velocities and then it reaches a plateau at a value which, in this case, does not seem to depend on the orifice size.

Going one step further, in Fig. 3(c) we report the probability density functions (pdfs) of $\mathbf{v}_z$ for different values of v_{belt} and compare them with the case of free discharge. We display results obtained for $D = 1.53$ cm, but the other orifice sizes explored are similar. At first sight, it is possible to distinguish between two kinds of pdfs. The first one, for large values of v_{belt}, consists on rather wide distributions, centered at nonzero values of $\mathbf{v}_z$ and only slightly asymmetric hence suggesting the existence of continuous flow. Within this group we can also frame the results obtained for the free discharge, even though these distributions evidence a small bump for $\mathbf{v}_z \approx -15$ cm/s [33]. The other kind of distribution appears

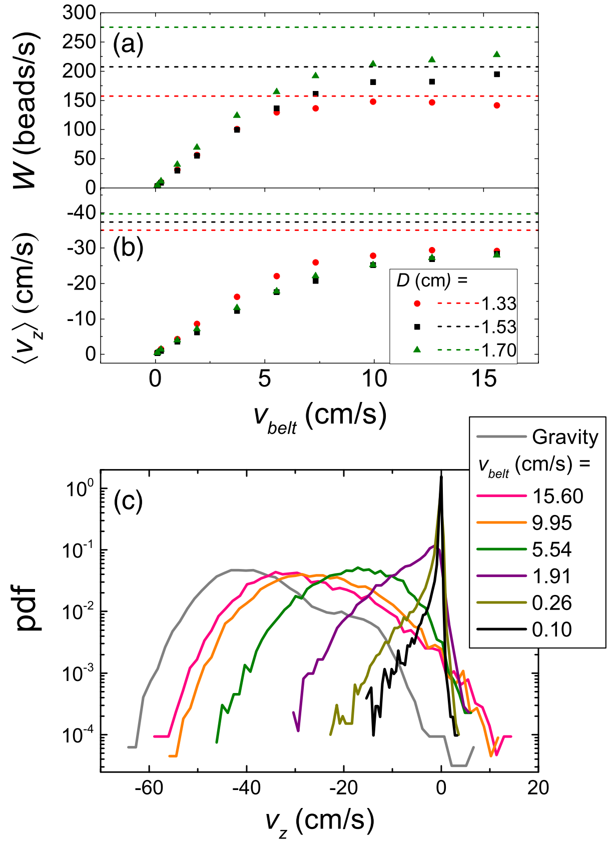

FIG. 3. (a) Mean flow rate W and (b) mean vertical velocity $\langle \mathbf{v}_z \rangle$ of the beads as function of the extraction belt velocity v_{belt} for the three orifice sizes indicated in the legend. The maximum v_{belt} investigated is given by the experimental limitations. The horizontal dashed lines indicate the values obtained for silos where the grains fall freely by gravity. (c) Probability density function (pdf) of $\mathbf{v}_z$ for an aperture size of $D = 1.53$ cm for the values of v_{belt} indicated in the legend and for the free fall discharge. Note the log-lin scale.

for small values of v_{belt} and is characterized by a narrow peak at $\mathbf{v}_z = 0$ which indicates the existence of an intermittent motion involving time intervals with the material at rest. Then, it can be stated that within the range of v_{belt} explored we move from a continuously flowing regime to an intermittent one. Despite the complexity of this flow, which undoubtedly should be further studied in future works, we will use the arithmetic mean value of the particles velocities $\langle \mathbf{v}_z \rangle$ as a first order control variable; in particular we use the absolute value of this magnitude defined as $v = |\langle \mathbf{v}_z \rangle|$. The appropriateness of this election is supported by the results obtained when changing the gap between the silo and the belt as it will be explained below.

Figure 4(a) shows the survival functions or complementary cumulative distribution functions (CCDF) of the avalanche size s for an orifice size of $D = 1.53$ cm and different values of v. The linear dependence of the CCDFs in log-lin scale agrees with a constant probability of clogging p_c over the whole avalanche duration. Indeed, p_c is defined as $1 - p$ (where p is the probability that a particle passes through the orifice without forming a clog) and can be extracted from the slopes of the CCDF [17]. Clearly, the higher the velocity of the grains the smaller the clogging probability and, then, the larger the avalanche size. This dependence is confirmed by plotting p_c versus v for all the orifices explored [Fig. 4(b)]. Essentially, this graph provides similar information to Fig. 2, but involves the convenient variables. A proof of the suitability of v as control variable has been reached by performing additional experiments with a different gap between the silo and the belt. This parameter considerably affects the particles velocities [33] but the clogging probabilities also change in such a way that they collapse on top of the p_c versus v curve [empty squares in Fig. 4(b)]. In addition, by implementing v as the control parameter, we confirm that, for the limited case of small velocities, the clogging probability is smaller than one (compatible with a finite value of the avalanche size) and its value depends on the outlet size.

Aiming to explore this limit case, we have carried out experiments with a very low belt velocity ($v_{\mathrm{belt}} = 0.1$ cm/s) and changing the orifice size. The results are presented as function of D with blue diamonds in Fig. 4(c). Clearly, the outcomes are compatible with an exponential of the outlet size raised to the problem dimensionality as proposed in [16–18]:

$$p_c = e^{-(D/d_p)^2 \ln a}, \tag{1}$$

where $a = 1.33$ is the only fitting parameter. The inclusion of $\ln a$ in the exponential term has been inspired by [18] and allows rewriting Eq. (1) as $p_c = a^{-(D/d_p)^2}$.

Based on this simple expression, in order to account for the kinematic effects reported in Fig. 4b, we propose to replace a by a linear ansatz: $a + bv$. Thus,

$$p_c = (a + bv)^{-(D/d_p)^2}. \tag{2}$$

This equation involves two fitting parameters: the aforementioned a, a parameter that determines the clogging probability in absence of inertial effects; and b, a parameter that establishes the influence of grains velocity on clogging. Noteworthy, all data sets in Fig. 4(b) can be fitted using $b = 0.0128$ (cm/s)$^{-1}$ in Eq. (2), keeping the previously obtained value of the parameter $a = 1.33$. Hence, the new expression proposed is able to predict the clogging probability when modifying the outlet size and/or the particles velocities, which contribution has been isolated and related with a single parameter each.

Now, the question to address is whether the same Eq. (2) serves for a silo where the grains fall freely under gravity, a scenario where the variation of the exit size implies altering the velocity of the grains. Remarkably, the answer is positive as evidenced by the collapse of the two series

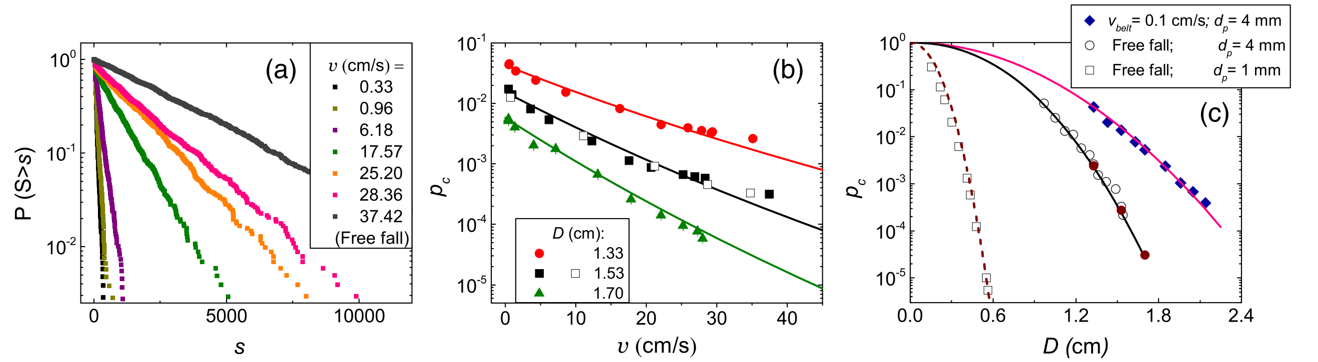

FIG. 4. (a) Complementary cumulative distribution functions of the avalanche size for $D = 1.53$ cm and the values of the mean vertical velocity showed in the legend. (b) Experimental clogging probability as function of the absolute value of the mean vertical velocity for the three orifice sizes displayed in the legend. Empty squares represent the values obtained with a gap of $h = 4.6$ mm, larger than the standard one ($h = 3.2$ mm). (c) Clogging probability versus the outlet size for a very low extraction velocity ($v_{\mathrm{belt}} = 0.1$ cm/s) and for gravity discharged silos with particles of 1 and 4 mm diameter as indicated in the legend. In both (b) and (c) the solid and dashed lines correspond to Eq. (2) with the parameters indicated in the text. In (c) the expressions for the gravity discharged silos are obtained using the expected grains velocity $v = \sqrt{gD}$. Also, the solid circles in (c) correspond to Eq. (2), but use the actual experimental velocity.

of data represented by circles in Fig. 4(c). On one side, we represent the clogging probability for $d_p = 4$ mm particles discharged freely from a silo (empty circles), and in the other side, we use the experimentally measured velocities of the grains in the free fall discharge and introduce them in Eq. (2) (filled circles). This result can be generalized for any outlet size by replacing in Eq. (2) the grains velocity $v = \sqrt{gD}$ [28,36] as represented by the continuous black line in Fig. 4(c). The agreement with the experimental data is excellent proving the validity of the proposed expression to describe clogging in a gravity discharged silo.

Going a step further, we challenge the validity of Eq. (2) by looking its applicability to reproduce the clogging of particles of the same material but considerably smaller diameter ($d_p = 1$ mm [17] instead of $d_p = 4$ mm). Interestingly, despite the important variation of the clogging probability with the particle diameter, a good correspondence is accomplished using Eq. (2) keeping $a = 1.33$ and reducing b to half its value: $b_{d=1} = b_{d=4}/2$ [dashed line in Fig. 4(c)]. We speculate that this could be attributed to a difference in the dissipation time that is required to completely cool down the system (which might be higher for bigger particles); this hypothesis should be confirmed in future works.

In summary, we introduce a new type of silo discharge method that allows decoupling the effect of the outlet size from the particles velocities in the development of clogging. From this setup, we propose a new clogging expression with only two fitting parameters. The first one determines the clogging probability when the grains velocities are minimized and could be related to plastic rearrangements of grains occurring when removing the particles (virtually one by one). Accordingly, the particle size does not seem to affect the value of this parameter, provided that the outlet size is modified proportionally. The other parameter, which accounts for the contribution of particles' kinematics, can be accessed when controlling the grains velocities by means of the extracting belt. Remarkably, this dependence of the clogging probability on particles' kinematics has remained hidden in experiments of gravity discharged silos. Nevertheless, this result agrees with some recent works that have suggested that the clogging process is affected by variations in the beads velocity, induced either by a narrowing of the silo [22] or by the material submersion into a fluid [27]. Therefore, after a suitable validation of our findings in (three-dimensional and large-scale) industrial silos, novel strategies such as tuning or modulation of belt velocity could be tested to improve granular flow. Our results are also specially valuable in the shake of establishing analogies among granular bottleneck flow and other physical systems (such as bubbles, colloids, animals, or pedestrians). This is so because most of these systems are characterized by being velocity controlled; i.e., there is no constant force driving the system but a limit velocity that the particles can reach.

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