Dynamical Patterns in Bénard-Marangoni Convection in a Square Container

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In Bénard-Marangoni convection in a square vessel, of small aspect ratio, a sequence of bifurcations is observed as control parameters are slightly changed. Some of the emerging patterns are stationary and others are oscillatory in nature. The stationary patterns break the symmetry and the oscillatory ones can be classified in three kinds. We show that this dynamics can be explained assuming that the system is close to a Takens-Bogdanov bifurcation.

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Pattern formation is an area of active research in a wide variety of extended physical systems. One of the most studied systems that presents this phenomenon is the Bénard-Marangoni convection, i.e., a fluid in a convective vessel heated from below with an upper free surface. This system organizes itself into convective cells, provided that the difference of temperature between the bottom plate and the free surface is beyond a critical ΔT_c. As in many extended systems, the role played by the boundary conditions is not completely understood. In order to investigate this aspect of pattern formation, we performed a Bénard-Marangoni convective experiment in a small aspect ratio vessel (small ratio between a typical horizontal length and the fluid depth). In this case, the boundary conditions are expected to determine the primary pattern that is selected. Single mode [1,2] as well as more complicated combinations of modes can be obtained [3]. In this Letter we report that some of these patterns can undergo secondary and tertiary bifurcations displaying oscillatory behavior. Moreover, one of these oscillations constitutes a dynamical version of one of the elementary topological processes observed in two-dimensional cellular patterns: side-swapping (also known as T_1 process), present in many different physical systems [4,5].

This dynamics is obtained in a container with square insulating lateral walls filled with a silicon oil of high Prandtl number (Pr = 3200). The experimental setup and the shadowgraph observation method are similar to the ones reported in [3]. We only recall here that the images recorded are obtained by a Schlieren technique. In these images the bright lines correspond to the cold parts of the pattern, i.e., descending motion of the fluid. The main experimental setup difference between the work in [3] and the present one is the use of square boundaries.

For the experimental parameters that we used (aspect ratio $\Gamma = 4.46$, temperature of the bottom plate between $T = 35\,^\circ\mathrm{C}$ and $60\,^\circ\mathrm{C}$, and ambient temperature near the free surface $T_a = 21\,^\circ\mathrm{C}$, which corresponds to Rayleigh numbers between Ra = 10 308 and Ra = 33 504) the system organized itself in four internal cells. In some cases the four cells were quadrilateral, and in others two of the cells were quadrilateral and the other two were pentagonal. As the temperature of the bottom plate was increased (the ambient temperature T_a was kept fixed), the following qualitative changes were observed. At threshold, the first convective pattern obtained was the one consisting of four square cells [see Fig. 1(b)]. A further increase leads to either the asymmetric pattern displayed in Fig. 1(a) or the one displayed in Fig. 1(c). These patterns are conjugated by a reflection with respect to an axis parallel to the walls. Under an additional increase of the temperature, these patterns do not remain stationary but begin to oscillate. We distinguish three kinds of oscillations; the first ones observed are periodic modulations of the length of the link between the two square cells of either Fig. 1(a) or 1(c). As the temperature is further increased a third kind of oscillation is found which consists of a periodic alternation between the pattern of Figs. 1(a) and 1(c) passing through the symmetric pattern of Fig. 1(b). The order of magnitude of the typical oscillation's period is one minute (several times smaller than the vertical thermal diffusion time). In order to check the stability of these phenomena, at each stage we run the experiment for at least one day, which corresponds in the case of the oscillations to several thousand periods. The experiment was repeated for a small inclination of the cell (i.e., the container slightly rotated around an axis parallel to two of the walls; X_1 in Fig. 2), and the phenomenology previously described persisted. These experiments have in common a reflection symmetry with respect to the X_2 axis.

In order to make a quantitative description of the experimental observations we have to choose appropriate variables. Let us define

$$x = d\cos(\alpha)\,, \tag{1}$$

with $\alpha \in (0,\pi)$, where d stands for the length of the diagonal segment and α stands for the angle between the segment and the X_1 axis in Fig. 2. The restricted domain of α reflects the fact that the patterns that we observed keep the symmetry $(X_1 X_2) \to (-X_1, -X_2)$ [6]. Moreover, these patterns can be described with only two values of α ($\alpha = \pi/4$ and $\alpha = 3\pi/4$); therefore the variable x describes the size of the link between the square cells and its inclination (left or right). As we have oscillations in the problem, we realize that a dynamical system that models

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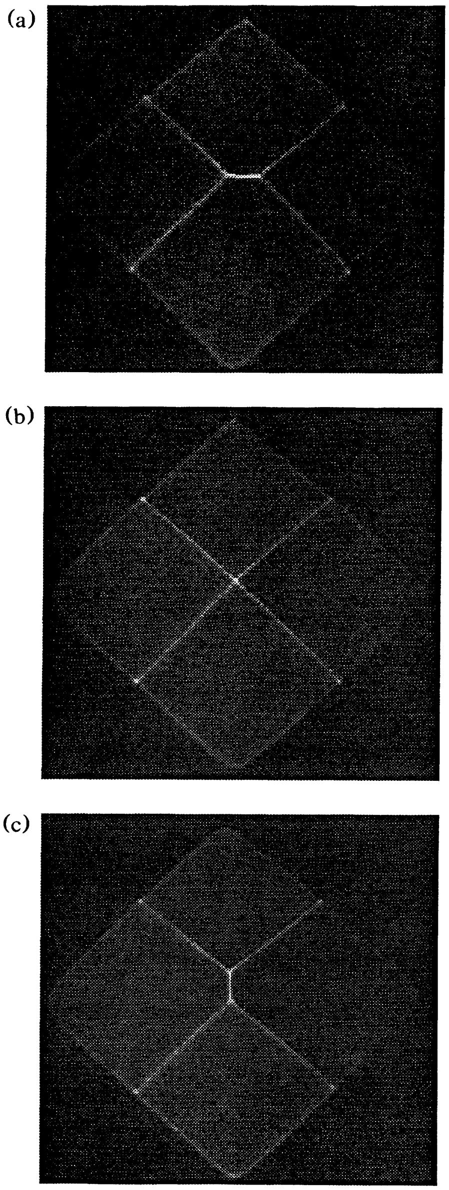

FIG. 1. Shadowgraph images of the convective patterns in a cell of aspect ratio $\Gamma=4.46$ (size of the cell, 68×68 mm^2), filled with silicon oil of viscosity 350 cS (1 cS$\equiv 10^{-2}$ cm^2/s). The bright lines correspond to the minimum of the temperature field, where the motion of the fluid is downwards. (b) corresponds to the symmetric pattern appearing at threshold, and (a) and (c) to the asymmetric ones. Those are born from the symmetric solution in a pitchfork bifurcation.

the phenomena previously described should have at least dimension 2. The time series data for these oscillations are displayed in Fig. 3. Performing an embedding (x,x') we obtain for the experimental data the reconstructed phase spaces displayed in Figs. 4(a)–4(d). This embedding successfully lifts self-intersections of the flow.

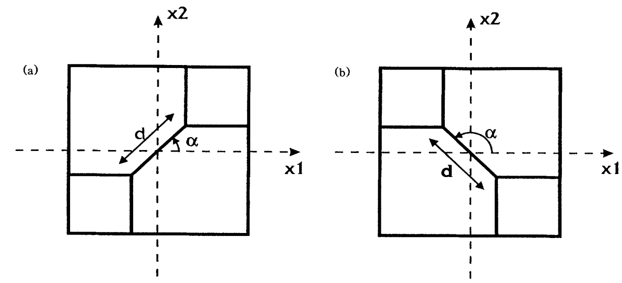

FIG. 2. Schematic representation of the asymmetric patterns, where d stands for the length of the diagonal segment and α is the angle between that segment and the X_1 axis. In our experiment $\alpha=\pi/4$ or $3\pi/4$. The variable x defined in Eq. (1) is thus positive in (a) and negative in (b). A symmetric pattern would be characterized by $x=0$.

The above observation suggests using the variables (x,x') as our dynamical variables, and a minimal dynamical model can be constructed

$$x'=y\,, \tag{2}$$

$$y'=f(x,y)\,. \tag{3}$$

As this model is to be equivalent under the reflection symmetry $x\to -x$ of our experimental setup (broken in the first stationary bifurcation that leads to the asymmetric patterns), the action of that symmetry on the variable y must be $y\to -y$ [according to Eq. (2)], and

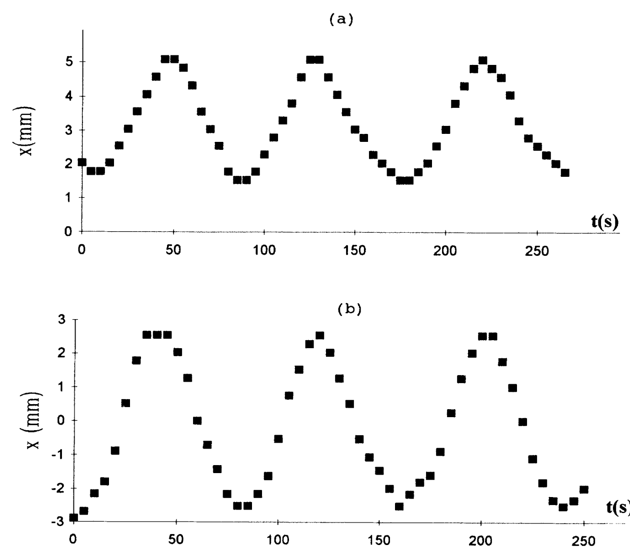

FIG. 3. Pieces of the time series data giving the evolution of the variable x as a function of time. The error in the x values (measured using an image processing code) is of about 0.2 mm. (a) shows a periodic modulation of x for a pattern similar to the one in Fig. 1(a) (aspect ratio $\Gamma=4.46$, temperature $T=57\,^\circ$C). (b) shows a periodic alternation between the two asymmetric patterns 1(a) and 1(c) ($\Gamma=4.46, T=59.8\,^\circ$C).

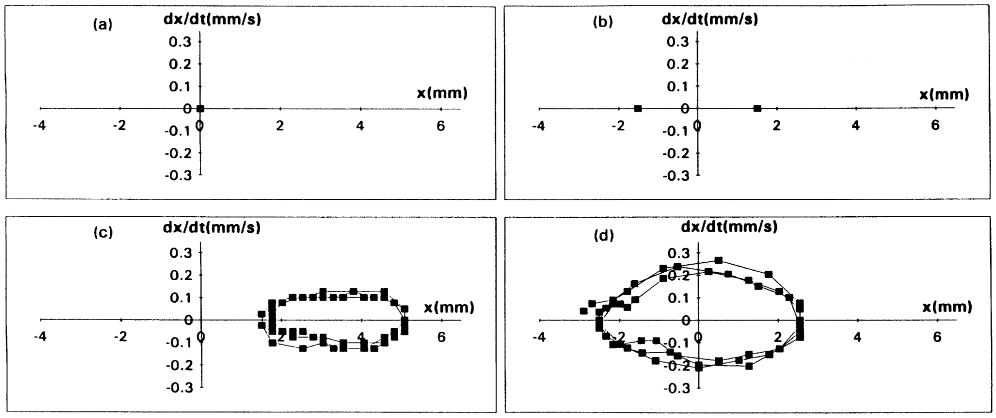

FIG. 4. Reconstructed phase space (x,x'). In (a) a fixed point in (0,0) corresponding to a stationary symmetric pattern ($\Gamma=4.46$, $T=35\,^\circ\mathrm{C}$). (b) shows the fixed points corresponding to the stationary asymmetric patterns ($\Gamma=4.46, T=45.6\,^\circ\mathrm{C}$). In (c) and (d) we display the phase space embeddings of the data presented in Fig. 3. The derivative was computed for the nth point as $(x_{n+1}-x_{n-1})/2\delta t$.

$f(-x,-y)=-f(x,y)$ [according to Eq. (3)]. Therefore, to third order

$$f(x,y)=ax+by+cx^3+dx^2y+exy^2+fy^3 . \qquad (4)$$

As in the experiment, stationary symmetry breaking bifurcations and Hopf bifurcations occur close in parameter space; the Jacobian of the vector field of Eq. (2) must have two eigenvalues close to zero. In terms of our model that implies a and b close to zero. These equations were first studied by Takens and Bogdanov [7,8], who showed that all of that family of equations can be reduced to two cases, according to the qualitative features of their solutions, namely, to

$$x'=y , \qquad (5)$$

$$y'=\mu_1 x+\mu_2 y \mp x^3+x^2 y , \qquad (6)$$

with μ_1,μ_2 small parameters. The solutions of these

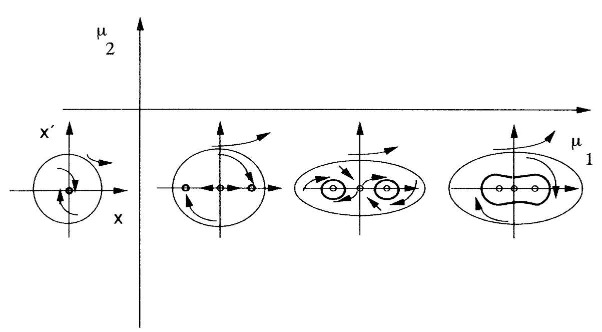

FIG. 5. Unfolding of the Z_2 equivariant Takens-Bogdanov bifurcation. Notice that if μ_1 increases monotonically with the temperature, the qualitative behavior observed in the experiment is reproduced by the solutions of Eqs. (5) and (6).

equations are displayed in Fig. 5 for the case with the minus sign. In region 1 of Fig. 5 there is an attractor at $(x,x')=(0,0)$, which corresponds to the stationary symmetric solution. In region 2, two solutions born in a pitchfork bifurcation are stable. These correspond to the stationary asymmetric solutions. In region 3, those solutions are unstable after undergoing a Hopf bifurcation in which two limit cycles are created. These solutions correspond to the asymmetric oscillations. Finally, for parameter values in region 4 a symmetric limit cycle (corresponding to the symmetric oscillations in the experiment) is born in a global bifurcation [7,8]. Notice that the one to one correspondence between the solutions of Eqs. (5) and (6) and the solutions of the experiment is achieved if the parameter μ_1 increases monotonically with the temperature of the bottom plate.

In this Letter we report a sequence of qualitative changes in the convective patterns of a square Bénard-Marangoni cell with small aspect ratio. This phenomenon can be organized assuming (1) that the experiment runs at parameter values close to a Takens-Bogdanov bifurcation and (2) the boundary conditions impose a reflection symmetry on the system which is translated into the model in the order of the nonlinearities in the normal form [9]. This opens an interesting way to investigate the connection between the dynamics of different secondary instabilities in the Bénard-Marangoni convection and the theory of dynamical systems.

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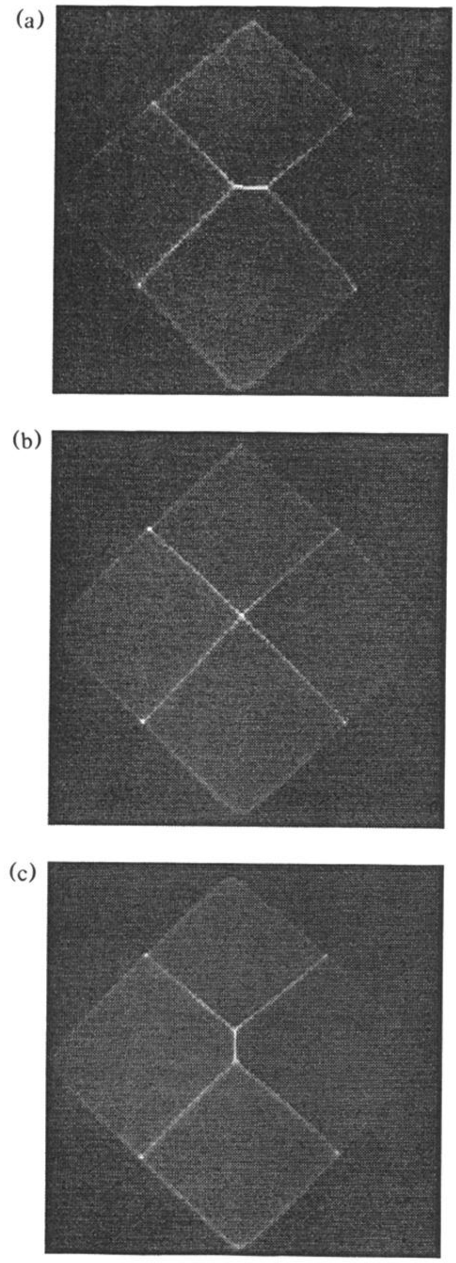

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