Testing the Kibble-Zurek mechanism in Rayleigh-Bénard convection

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We report experimental evidence of the fact that, in an emerging Rayleigh-Bénard structure, the density of defects which appear scales as a power law in the rate of change of the control parameter. The scaling exponents agree with those calculated from the Kibble-Zurek mechanism. This is the first evidence to our knowledge that this mechanism works in a hydrodynamical system.

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Transitions in nature where there is a symmetry breaking are ubiquitous: from the Big Bang, after which soon appeared huge cosmological structures like galaxies as well as the tiny elementary particles [1], to bifurcations between nonequilibrium states in many-body systems [2], including many condensed matter phase transitions like the martensitic, superconducting, and superfluid transitions among others [3–5].

One of the most interesting features of symmetry breaking transitions is the evolution of the system through the symmetry breaking process.

The behavior of out-of-equilibrium bifurcations in continuum media is characterized by emerging macroscopic structures commonly called patterns [6,7] which can be characterized by a generalized order parameter field. The main advantage of studying those systems as models for all symmetry breaking transitions comes from their ease of analysis and experimental treatment due to the human scale of the structures formed.

Usually the appearance of phase singularities or topological defects is involved with such transitions in the following way. Consider a symmetry breaking transition, that is to say, a transition from a more symmetric phase to a less symmetric one. First, topological defects could arise as a relic of the more symmetric phase in the less symmetric one, when crossing from the former to the latter. Second, these defects are topologically stable. The core of the defects is a localized state different from the (stable) less symmetric phase. This localized state is stabilized by topological constraints, which also diminish its symmetry. Consequently the defect represents a local bifurcated state with a lower symmetry than the less symmetric phase [8,9]. On the other hand, when the control parameter is increased enough in the same phase, there appear defects as a result of the nonlinearities in the system. These defects, like the others described before, also are local bifurcated states with less symmetry. In the next symmetry breaking transition these defects could be the germs from which the next stable state emerges [8,9]. In conclusion, defects play a major role in the transition from one symmetry to another, actually allowing the transition [10]. Topological defects in structures can be classified according their homotopy group [11–15].

Kibble [1] proposed that defects also are important in the phase transitions occurring soon after the Big Bang because they yield the density of galaxies. After him, Zurek [16,17] extended this kind of mechanism of defects appearing to condensed matter systems. The argument is the following. Suppose that the control parameter is changing linearly slowly. If the system is far from the critical point, the state changes adiabatically to fit to the equilibrium state defined by the instantaneous control parameter value. But, when the system is near a second-order transition, the equilibrium correlation length should change more quickly than the limiting speed in the system due to the slowing down of the relevant modes. This speed, in condensed matter systems, is commonly the speed of sound. Thus, the system gets frozen until the adiabatic dynamics is restored well after the transition point. The correlation length measured after the transition is the one the system had when it froze. This is known as the Kibble-Zurek mechanism, and is thought to lead to a universal scaling law for the correlation length upon the rate of change of the control parameter, which depends only on the space dimension, topology, and dissipative character of the system.

This point of view is compatible with what has been said above. A symmetry breaking transition where two equivalent domains could grow in the more symmetric phase may lead to a lack of "phase matching" for the less symmetric domains. The phase is the parameter(s) corresponding to the symmetry that is broken in the transition and could lead to two (or more) equivalent domains. The localized state with even fewer symmetries appearing in the region where the lack of phase matching occurs is a topological defect. These topological defects appear with distances among them that give, on average, the correlation length of the order parameter.

The possibility of confirming cosmological theories in a laboratory [16,18–20] has induced the performance of several experiments on nonequilibrium phase transitions, among others in superfluid helium [21–27], liquid crystals [28–30], superconductors, and Josephson junctions [31–36], and in nonequilibrium bifurcations in nonlinear optical systems [37,38] and convective systems [38–40].

In this Brief Report we aim at verifying that there exists a scaling law for the defect density when the rate of change of the control parameter is swept across the symmetry breaking transition point, focusing on the critical exponent of such a scaling law. The purpose is to shed light on the question of whether the Kibble-Zurek mechanism could work for hydrodynamic systems or not, considering the negative results in

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the past [39]. In that work, the discrepancy was suggested to be attributable to the fact of more than one interacting mode coexisting in the less symmetric phase.

Here we report experimental results concerning the defect density which appear in a conduction-convection bifurcation of a Rayleigh-Bénard system [41–44]. The state below the bifurcation threshold is a homogeneous conduction state. The state after the transition point comes from the destabilization of convective modes, and may lead to a structure of stripes (convective rolls) or squares (convective cells) depending, among other parameters, on the thermal conductivity of the boundaries relative to the fluid. Our experimental set-up allows us to check the Kibble-Zurek mechanism in a Rayleigh-Bénard system [17,45] with two convective modes, while the previous work in nonequilibrium bifurcations [37–40] explored the one-mode case (which agreed with Zurek-Kibble theory) and the three mode case (which did not agree with that theory).

For experimental studies, a layer (depth $h=3$ mm) of silicone oil with $\nu=20$ cS is placed in a nylon square container. Below the fluid there is a polished metallic plate heated from below by an electric resistance designed for a homogeneous distribution of heat. The upper surface of the fluid is in contact with a glass window, whose temperature $T=T_0$ is stabilized by a thermal bath.

In the range of applied temperatures the physical properties of the fluid do not suffer a great change (the Boussinesq approximation is valid). It is transparent to the light, allowing optics measurements. Furthermore, in the region of control parameter used there is only a primary bifurcation.

Local temperature was measured by three T-type thermocouples (see Fig. 1), the first one just below the fluid and the other two at the cell entrance and exit of the water used by the thermal bath. The reading of thermocouples is done by a computer-controlled multimeter. The global temperature field [pattern, see Fig. 1(b), right] is obtained by a shadowgraph and structure images are captured by a charge-coupled device camera connected to a computer.

The experimental setup described above is similar to others used by our group before [38–40].

The control parameter of Rayleigh-Bénard convective systems is $\epsilon=(\Delta T-\Delta T_c)/\Delta T_c$, which means the nondimensional distance to the (static) convective transition point. The measurement process consists of the following steps. First the system is set in a stationary conductive state ($\epsilon=-\epsilon_0$) [see Fig. 1(b), left and bottom] just below the (static) convective threshold, by applying a power P_1 to the heater. The measurement begins with a sudden increase of the power delivered to the system to a value P_2 [see the curve in Fig. 1(b), top]. The control parameter ϵ increases, at the considered time scale, linearly [see Fig. 1(b), bottom] in time. The times are nondimensionalized with the vertical temperature diffusion time. At some time corresponding to $\epsilon=\hat{\epsilon}>0$ a structure abruptly gets formed, and then a snapshot of the pattern is taken to analyze it [45]. We define the rate of increase μ of the control parameter as the nondimensional slope of the control parameter at $\epsilon=0$. The first run of this procedure allows us to determine the value of the control parameter when the structure is formed, $\hat{\epsilon}$, at a fixed μ. Using this value as the reference when the snapshot has to be

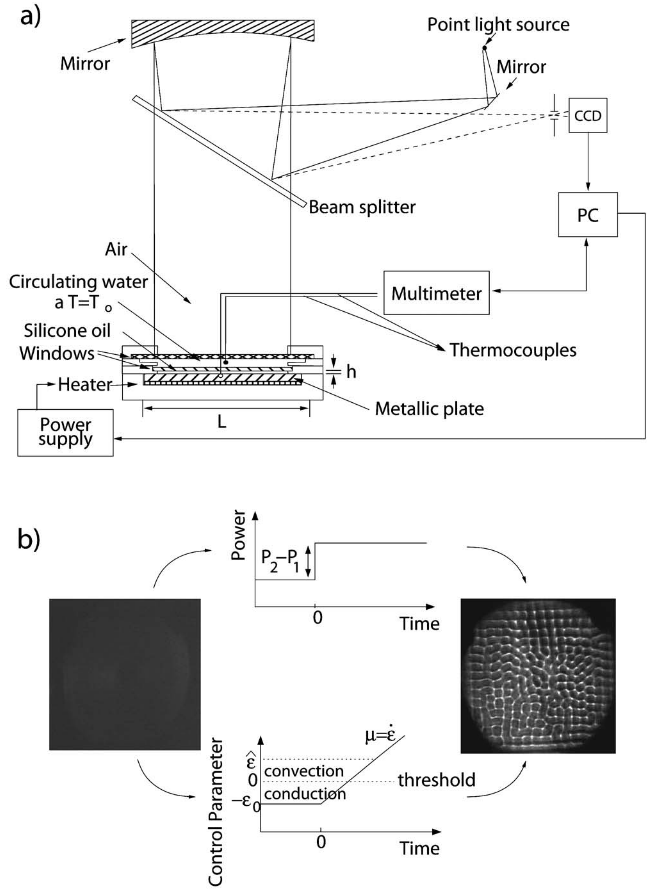

FIG. 1. The experimental setup.

taken, the procedure is repeated ten times for each sudden increase of power delivered to the system, as a compromise between the time spent on the experiment and the statistical error. After that, the whole process is repeated for another μ, which can be obtained changing the value of P_2.

The image obtained in each measurement [similar to the image shown in Fig. 1(b), right] is processed. Since the patterns obtained are of square geometry, it is not possible to proceed to study the connectivity properties of the convective cells through Voronoi analysis (as in [39]). The problem is the geometric instability of the coordination number for this kind of pattern, i.e., in a perfect square pattern if one site is very slightly moved (in such a way that clearly does not appear as a topological defect) the coordination number for this site changes abruptly. Here, instead, we use the method of complex demodulation [40,46,47]. As the square pattern can be seen as the superposition of two perpendicular modes, whose directions are approximately anchored by the boundaries, we consider these modes filtered with the largest possible radius without overlapping. In this way we obtain the topological defects as phase discontinuities. Also, we reject the region close to the boundaries to remove their influence on the correlation properties of the phase.

It is important to say that the filtering in the Fourier space inserts a coarsening spatial scale. This problem becomes important only for defect densities greater than $1/\lambda^2$, where λ is the characteristic wavelength of the pattern. This is not the case obtained in this work.

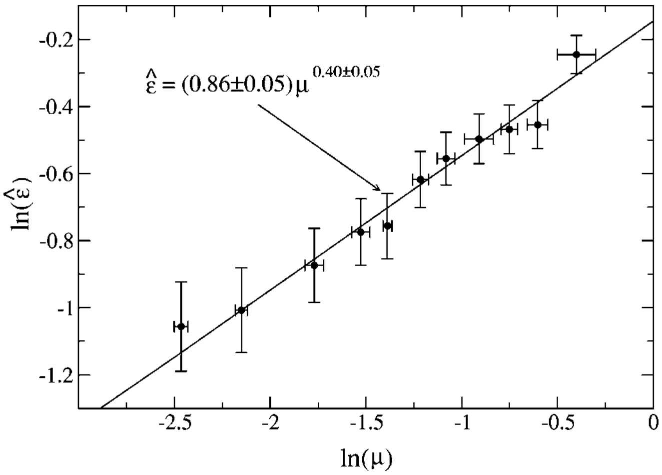

FIG. 2. Value of the control parameter at which the structure is formed ($\hat{\epsilon}$) as a function of μ. Both axes are on logarithmic scale.

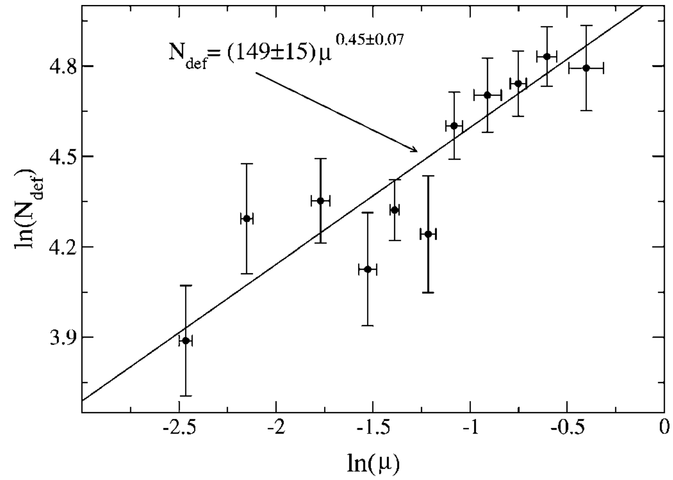

FIG. 3. Number of defects (at $\epsilon=\hat{\epsilon}$) vs μ. Both axes are on logarithmic scale.

We report how the control parameter value at which the structure forms, $\hat{\epsilon}$, depends on the rate of increase of the control parameter (μ), and also how the number of defects (related always to the same area) at the time corresponding to $\epsilon=\hat{\epsilon}>0$ depends on μ. The physical aspect ratio $a=L/\lambda$ is the nondimensional length of the cell side. In this experiment it is 19.

There is a limit to the range of possible time scales to choose μ. On the side of fast quenches the experiments are basically limited by the thermal inertia of the metallic plate (of the order of 15 s) and by the vertical thermal diffusion time—defined as the square of the fluid depth divided by the thermal diffusivity—which is 91 s. On the side of slow quenches, we are limited by the slow dynamics of the structure due to small inhomogeneities in the system [48].

In all the measurements the number of defects obtained is larger than 50, usually being around 90. This fact, together with the multiple (10) sudden increases in power delivered to the system, warrants that we can extract scaling law properties from the results, as the statistics is good enough.

In Fig. 2 we report results of $\hat{\epsilon}$ versus the rate μ in logarithmic scale. Each curve point is the mean of ten measurements, and the error bars on the two axes are the standard deviations of the control parameter of appearance of the structure and of μ. The curve was fitted well with a power law and the exponent is 0.40 ± 0.05. Figure 3 reports the number of defects when $\epsilon=\hat{\epsilon}$ versus the rate μ on a logarithmic scale. The curve was fitted with a power law, as shown in Fig. 3.

As the system area is constant, as well as the number of convective cells, the curve of the number of defects (Fig. 3) scales in the same way as the density of defects ρ_{def}.

The first thing to check in order to verify the Kibble-Zurek mechanism is that the control parameter corresponding to the formation of the structure ($\hat{\epsilon}$) follows a power law with its rate of change (μ). Comparing with the work of Zurek [17] the exponent of the curve should be $\frac{1}{2}$. As the snapshots are taken at a fixed $\hat{\epsilon}$ for each power delivered to the system, the measured rates of change of the control parameter are lower bounds of the expected value, due to the overestimation of the value of $\hat{\epsilon}$ because of the experimental system resolution. Furthermore, this overestimation is higher at lower values of μ. So we expect that the measured value for the exponent (0.40 ± 0.05) is slightly smaller than the one predicted by Zurek.

Regarding the result for the number of defects, the exponent predicted by Zurek is also $\frac{1}{2}$ [17]. This value lies within our experimental error (0.45 ± 0.07).

In conclusion, the experimental results presented here show that the number of defects and the control parameter value at the appearance of structure vs the quench rate follow scaling laws for a conduction-convection transition in a Rayleigh-Bénard system. The values of the exponents are compatible with the ones predicted by Zurek for condensed matter systems.

The agreement of the exponents confirms that there are hydrodynamic systems with more than one mode coexisting in the less symmetric phase in which the Kibble-Zurek mechanism works. The disagreement with previous negative tests [39] might be because in the kind of system studied there (Bénard-Marangoni) the interaction between the different modes in the transition is very important [40], while in the system studied here such interaction is not important. Another possible reason could be related to the fact of having more than two modes intrinsically.

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