A Discrete Spheropolygon Model for Calculation of Stress in Crowd Dynamics

Fernando Alonso-Marroquín, Jonathan Busch, Alvaro Ramírez and Celia Lozano

Abstract Several models have been presented to evaluate flow rates in pedestrian dynamics, yet very few focus on the calculation of the stress experienced by pedestrians under high density. With this aim, a pedestrian dynamics model is implemented to calculate the stress developed under crowd conditions. The model is based on an extension of a granular dynamics model to account contact forces, ground reaction forces and torques in the pedestrians. Contact stiffness is obtained from biomedical journal articles, and coefficient of restitution is obtained by direct observations of energy loss in collisions. Existing rotational equations of motion are modified to incorporate a rotational viscous component, which allows pedestrians to come to a comfortable stop after a collision rather than rotating indefinitely. The shape of the pedestrian is obtained from a bird's eye, cross sectional view of the human chest cavity and arms, which was edited to produce an enclosed shape. This shape is them approximated by a *spheropolygon*, which is a mathematical object that allow real-time simulation of complex-shape particles. The proposed method provides real benefits to the accuracy on particle shape representation, and rotational dynamics of pedestrians at micro-simulation level, and it provides a new tool to calculate the risk of injuries and asphyxiation when people are trapped in dense crowds that lead to development of high pressure.

Fernando Alonso-Marroqun
School of Civil Engineering, The University of Sydney, Australia
e-mail: fernando.alonso@sydney.edu.au

Jonathan Busch
School of Civil Engineering, The University of Sydney, Australia

Alvaro Ramirez
Universidad Politécnica de Madrid, Australia

Celia Lozano
Departamento de Física, Facultad de Ciencias, Universidad de Navarra, Spain.

1 Introduction

Since its inception in the 1950's [1], numerous methods have been developed to best describe pedestrian behaviour, including the Social Force Model, cellular automaton, as well as fluid and gas representations. The study of pedestrian panic, however, began later in the 1990's in response to many of history's horrific crowd stampedes [2]. Typically pedestrian panic events occur in crowds with high density, at points where flows of people conflict with one another, or at doorways and corridors where the flow is constrained. Despite improvements in crowd management, 2,000 deaths annually occur in incidents owing to crowding. Hsieh *et.al* study considered 215 reports on stampedes occurred at sporting, musical, political and religious [3]. It was found that panic events at religious events were the most severe, with an average of 12 fatalities per stampede. Fewer fatalities are reported in political, sport and musical events, which produce 6, 5 and 3 deaths per stampede. The Muslim pilgrimage, or Hajj, to Mecca is a site with a long history of deaths due to stampedes resulting from panic events. Over the past three decades, nearly 3,000 people have been killed in stampedes during the Hajj; the one in 2006 causing at least 345 fatalities [4].

The numerical study of pedestrian panic has arisen because traditional methods did not attempt to describe the collision or interaction forces between pedestrians. Pedestrian panic models can quantify the interaction forces and stress distributions as pedestrians come into contact with one another. The model proposed in this paper is based on a Discrete Element Model (DEM) developed for granular materials. This approach takes advantage of the computationally efficient and accurate calculation of the interaction forces provided by the well developed study of granular material. The computer simulations in this model will be based on a complex-shapes granular method, which use spheropolygons instead of rigid discs as discrete elements. This allows for a more accurate pedestrian shape that reflects the morphology of the human chest. The behavior of pedestrians in this model will be governed by a modified form of the DEM which will include a driven force, collision forces, rotational equations of motion, as well as friction and viscous forces.

2 State-of-the-art of pedestrian dynamic modeling

Pedestrian dynamic modelling simulations have been developed since the pioneer work of Helbing who have been responsible for numerous advances in the field from the mid to end 1990s until now [5, 6, 7, 8, 4]. During this time, the key challenge in the pedestrian dynamic modelling has been to achieve rules that guide how the agents considered particles in many cases interact with each other in a way that faithfully reproduces behaviors commonly observed in reality. Earliest studies suggested that motion of pedestrians can be described as if they would be subject to the so-called social forces [5]. These forces would not be directly exerted by the pedestrians' personal environment, but they would be a measure for the internal motivations of the agents to perform movements. The social force model of Helbing and Molnar [5] is one of the best known approaches to simulate pedestrian motion, a collective phenomenon with nonlinear dynamics. The social force model considers that the Newtonian laws of motion mostly carry over to pedestrian motion. Therefore, human trajectories can be computed solving a set of ordinary differential equations for velocity and acceleration. Earliest studies of evacuation [9, 10] were based on a lattice gas model of pedestrian flows. Many observations of the dynamics of pedestrian crowds have been successfully described by simple many-particle models. Xia et al. [11] developed a macroscopic model for pedestrian flow using the dynamic continuum modelling approach. Chraibi et al [12] introduced a spatially continuous force-based model for simulating pedestrian dynamics, Guy et al. [13] proposed a simple and intuitive formulation based on biomechanical measurements and the principle of least effort. Baglietto and Parisi [14] presented an off-lattice automaton for modelling pedestrian dynamics. Here, pedestrians are represented by disks with variable radius. More recent models such as Alonso-Marroquín et al. [15] accounts for particle shape using spheropolygons and calculate the full visco-elastic forces.

3 Methodology

The proposed model is based on a modified DEM model that reflects the properties of pedestrians in dense crowds. The so-called spheropolygons are used to represent particle. A spheropolygon is the object generated by sweeping a disk along a polygon [16]; contact forces between spheropolygons are calculated from the vertex-edge distance between their polygons [17]. By including ground-reaction forces and torques, one can simulate dynamics of pedestrian using spheropolygons [15]. These studies represent granular flow in two dimensions, which is convenient to adapt to pedestrians, whose interactions with obstacles and other agents can generally be simplified to two dimensions.

In adapting granular force to pedestrians, the typical gravity force was taken to be analogous to the pedestrian driven force which is a subset of the social forces proposed by Helbing [18]. The agents' desired speed is equivalent to the terminal velocity of particle immersed in a viscous fluid. The crosss-section of a pedestrian was derived using an image of a cross section of human thorax and arms. The image was scaled to reflect typical sternum width, and simplified into a closed curve which is them approximated by a spheropolygon, see Fig. 1. The interaction includes contact forces, and self-driven forces, and torques accounting shoulder rotation. A visco-elastic force analogous to the granular force is applied at each physical contact between pedestrians. This force consists of an elastic spring and a damper that accounts restitution after collision [15]; The self-driven force accounts the desired walking speed in a direction chosen by the pedestrian. The motive torque is used by the agent to rotate from their current to their desired orientation was derived using methods proposed by Korohonen et al. [19] and Langston et al. [20]:

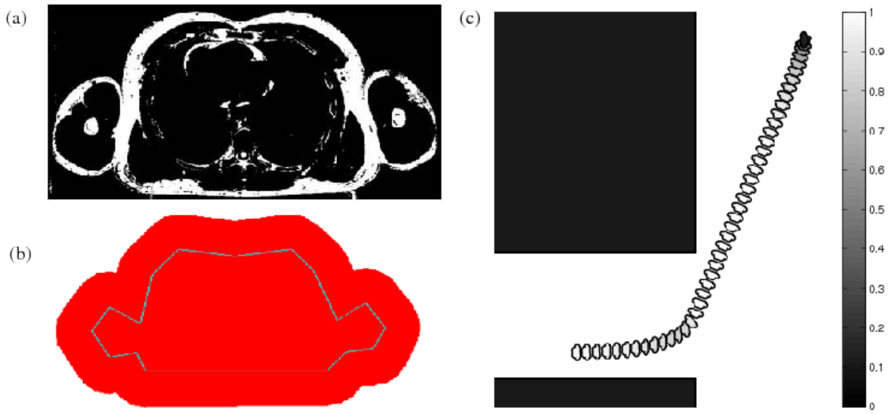

Fig. 1 (a) Boundary of the cross-section chest derived from US National Library Image of Thorax; and (b) representation of the thorax shape using a spheropolygon with 18 vertices and a spheroradius of 0.05 m. (c) Time lapse snapshots of the pedestrian displacement of reorientation. The colorbar encodes velocity [m/s]. the frame rate is 0.25 s.

$$T = \frac{\lambda I}{\tau^2}[1 - exp(-\frac{v}{v_0})](\theta - \theta_D) - \frac{I}{\tau}\omega \quad (1)$$

The first term is the ground reaction torque that was considered to arise solely from the pedestrian's desire to face toward their preferred destination. θ is the orientation of the pedestrian and θ_D is the angle to the walking direction. τ is the relaxation time of rotation, and I is the moment of inertia. The constant λ in Eq. 1 is a dimensionless constant that is derived by comparison of simulations with real pedestrian rotation. The exponential factor is used to account for lower motive torque at lower speeds. This is a reflection of higher rotation observed in pedestrians at lower velocities. In this factor we include the pedestrian speed v and the terminal speed v_0. The second term in Eq. 1 allows the pedestrian to reach their desired angular velocity was-. As pedestrians prefers not to rotate, the desired angular velocity was set to zero. As a consequence, the pedestrian comes to a comfortable stop after a collision rather than continuing to rotate.

The parameters of the contact force model were chosen as follows, see Table 1: the thorax responses to forces are taken from medical data on thorax deformation [21]; the coefficient of friction $\mu = 0.4$ is close to the coefficient of friction between pieces of cloth fabric. Since little information is available regarding the coefficient of restitution between pedestrians, during visual observations we obtained coefficients of restitution within the range $0.1 - 0.5$. The parameters of the torque were obtained from video footage observation, as explaned in the Table 1. For the simulation presented here, each pedestrian has an initial acceleration of $1m/s^2$ and a terminal velocity if $v_0 = 1m/s$.

The in-house object-oriented computer program SPOLY was used to conduct the simulations. SPOLY simulates the pedestrian dynamics based on spheropolygons and a five-order predictor-corrector numerical integration [20]. The use of a neighbour table and Verlet distances allows real-time simulation with up to 400 pedestrians. For large-scale simulations - with around ten thousands pedestrians - the algorithm executes around one minute of simulations per hour. Details relating to the SPOLY code are found in [16, 17, 22].

Table 1 Summary of the model parameters and agent properties.

Parameter	Name	Value	Obtained from
k_n	Normal stiffness	$10^5\ N/m$	Force-displacement relation
ρ	Agent Density	$10^3\ kg/m^2$	Average mass divided by average area
τ	Reaction Time	$0.5\ s$	Time required to reach the desired speed
μ	Coefficient of friction	0.4	Tables of friction between clothes
ε	Coefficient of restitution	0.3	Observations of energy loss after collisions
λ	Torsion stiffness	25	Comparison of simulation with video footage

4 Results

On the first test, we present the movement of a single pedestrian, as shown in the Figure 1. Initially, the pedestrian is facing west and its desired direction of motion is towards the centre of the exit to a hallway. Initially, the pedestrian rotates their shoulders parallel to its desired direction of motion. Then, the pedestrian accelerates until it reaches its desired speed. When the agent enters the hallways region, the pedestrian turns to east. Here we notice a slight deceleration, as the desired direction is not smooth at the exit. The deviation was not significant relative to forces observed under panic conditions.

Using these results, we simulate the evacuation of a room, and calculate the severity of the contact forces acting on the pedestrians during the process. A total of 500 agents were used in simulations. The room's dimensions were set to 20 m x 24 m, with the exit width varying between 1 and 3m. All widths lead to a bottleneck forming. As expected, the flow rate increases as the width is increased. When the exit is 1m width, the system clogs due to interlocking between chest and arm arrangements. Interestingly, this clogging seems to be stable to small perturbations such as slightly shaking of the pedestrians. In reality, we expect the pedestrians will be able to break that arch that is formed on the exit, but breaking is not an instantaneous process, and stresses due to contact forces will persist over minutes during evacuation. The contact forces create an intricate networks as shown in Figure 2. Each link in this network is constructed by connecting the centre of mass of the pedestrian with the contact point; the thickness of the width encodes the magnitude of the contact force. We separate the contact forces into severe $F > 400\,N$, mild $40\,N < F < 400\,N$, and low $F < 40\,N$. Severe forces are observed in all simulations. Intuitively, large forces remain for short time in wide corridors, while they are long-lasting in small ones. These long-lasting severe forces are more likely to produce injuries by asphyxiation, especially besides walls and near to the exits. Understanding how the contact network evolves in pedestrian dynamics is a key factor for calculating probability of injuries in crowd.

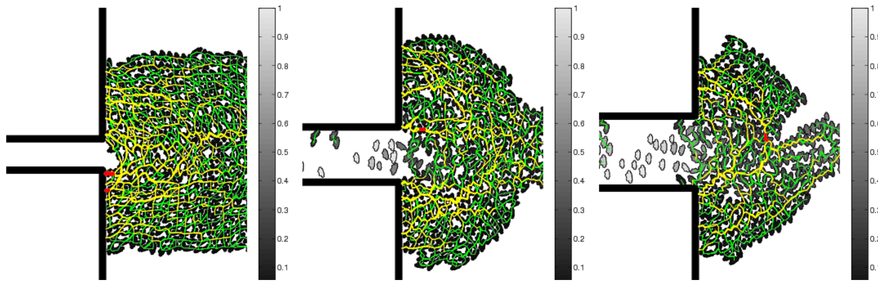

Fig. 2 Snapshot of crowd configuration after one minute of evacuation of a room. The width of the exit is 1m, 2m, 2.5m and 3m. The lines encode the magnitude of contact forces that are classified as severe (red) mild (yellow) and low (green). The colour of the pedestrians encodes their speed.

The final simulation prepared was the circling of the Kaaba in the muslin Hajj pilgrimage to Mecca, Saudi Arabia. This is the largest mass gathering in the world. The Kaaba is a cube shaped building that act as the direction of muslin prayer. In the Grand Mosque, the pilgrims circle seven times counterclockwise. In the preliminary simulation of this event, we represent both the Kaaba and the Hatim (the semi-circular wall besides the Kaaba) by fixed spheropolygons, see Fig. 3. A total of 6,800 pilgrims were distributed around the Kaaba. The desired direction of motion of a pedestrian is given by the vector $\mathbf{e} = \mathbf{k} \times \mathbf{r} - 0.1\mathbf{r}$; where $\mathbf{r}$ connects the centre of the Kaaba to the centre of mass of the pedestrian, and $\mathbf{k}$ is a unit vector perpendicular to the floor, pointing upwards. The first component of this equation accounts for the circulation of the pilgrim; and the second one accounts for the tendency to get approach the Kaaba.

Fig. 3 shows two snapshots at an early and advanced stage of simulation. The spiral-like pattern results from the combined rotation and attraction of the pilgrims around the Kaaba. Note that the Hatim acts an obstacle that keeps the pilgrims away from the Kaaba. After few minutes, the simulation reach a stationary stage as shown in Fig. 3 b. The Hatim helps to create ordered lanes where the pedestrian are in a comfortable contact with each other, These lanes break down spontaneously after a certain point, leading to a less dense configuration where the pedestrian are almost in no contact. The stresses developed in the circling of the Kaaba are quite low (below $F = 40\ N$), which indicate low probability of injuries under this ritual, which is consistent with the lower levels of injuries observed in this ritual. It is expected that major risks of fatalities are likely in the corridor and hallways used by the pilgrims to complete the whole Hajj ritual, and should be investigated in future research.

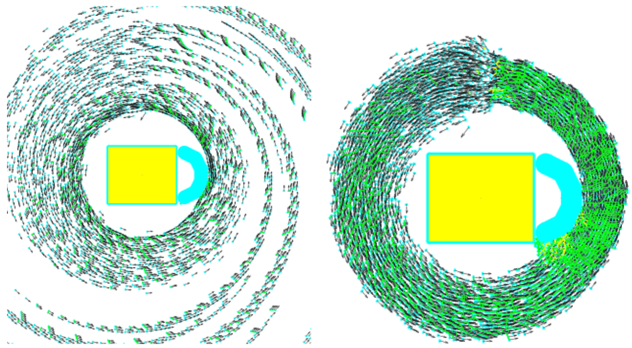

Fig. 3 Snapshots of the simulation of the circling of the Kabba in the Muslin Hajj peregrination. Besides the rectangular box (Kabba) lies the semi-circular wall (Hatim). The arrows represent the velocity of pedestrians. The snapshots are taken after 5 and 10 minutes.

5 Conclusions

The extension of the DEM spheropolygon method to simulate pedestrian dynamics provided an improved model in terms of the rotational motion, pedestrian shape and interaction via contact forces. The rotational equations of motion produced behavior that is consistent with pedestrian rotation. Contact parameters yielded a typical collision with a low coefficient of restitution which was indicated by research, and the shape improvements allowed for a more accurate description of the packing arrangements of panic disasters. The main difficulties with implementing the model were the inherent limitations from using a granular model with insufficient capacity to capture route choice behavior and rotation that precedes motion. These restrictions did not, however, obscure the benefits of the proposed improvements to simulate escape dynamics under panic situations. With this aim, we are calling for real case scenarios of crowd dynamics to be simulated with our in-house software SPOLY. We have capabilities to model real geometry based on scale plans. Besides, complex behavioral rules of the pedestrian can be introduced via ground reaction torques and forces. In addition, more customized pedestrians could be modeled including those that deviate from the crowd's speed to determine the behavior of a more complex crowd. Finally, a more general implementation of the crowd dynamic could be included to increase pedestrian avoidance at low speeds so that behavior is more streamlined and efficient. The field of pedestrian dynamics still has many opportunities for advances in the ever on-going pursuit of accurate modeling methods that inform safe design practices and descries crowd behavior.

References

1. BD Hankin and RA Wright. Passenger flow in subways. *OR*, 9(2):81–88, 1958.
2. Saad AH Algadhi and Hani S Mahmassani. Modelling crowd behavior and movement: application to makkah pilgrimage. *Transportation and Traffic Theory*, 1990:59–78, 1990.
3. Yu-Hsiang Hsieh, Ka Ming Ngai, Frederick M Burkle, and Edbert B Hsu. Epidemiological characteristics of human stampedes. *Disaster Medicine and Public Health Preparedness*, 3(04):217–223, 2009.
4. Dirk Helbing, Anders Johansson, and Habib Zein Al-Abideen. Dynamics of crowd disasters: An empirical study. *Physical review E*, 75(4):046109, 2007.
5. Dirk Helbing and Peter Molnar. Social force model for pedestrian dynamics. *Physical review E*, 51(5):4282, 1995.
6. Dirk Helbing, Illes Farkas, and Tamas Vicsek. Simulating dynamical features of escape panic. *Nature*, 407(6803):487–490, 2000.
7. Dirk Helbing, Rui Jiang, and Martin Treiber. Analytical investigation of oscillations in intersecting flows of pedestrian and vehicle traffic. *Physical Review E*, 72(4):046130, 2005.
8. Dirk Helbing, Anders Johansson, Joachim Mathiesen, Mogens H Jensen, and Alex Hansen. Analytical approach to continuous and intermittent bottleneck flows. *Physical review letters*, 97(16):168001, 2006.
9. Dirk Helbing, Motonari Isobe, Takashi Nagatani, and Kouhei Takimoto. Lattice gas simulation of experimentally studied evacuation dynamics. *Physical Review E*, 67(6):067101, 2003.
10. Motonari Isobe, Dirk Helbing, and Takashi Nagatani. Experiment, theory, and simulation of the evacuation of a room without visibility. *Physical Review E*, 69(6):066132, 2004.
11. Yinhua Xia, SC Wong, and Chi-Wang Shu. Dynamic continuum pedestrian flow model with memory effect. *Physical Review E*, 79(6):066113, 2009.
12. Mohcine Chraibi, Armin Seyfried, and Andreas Schadschneider. Generalized centrifugal-force model for pedestrian dynamics. *Physical Review E*, 82(4):046111, 2010.
13. Stephen J Guy, Sean Curtis, Ming C Lin, and Dinesh Manocha. Least-effort trajectories lead to emergent crowd behaviors. *Physical Review E*, 85(1):016110, 2012.
14. Gabriel Baglietto and Daniel R Parisi. Continuous-space automaton model for pedestrian dynamics. *Physical Review E*, 83(5):056117, 2011.
15. Fernando Alonso-Marroquin, Celia Lozano, Alvaro Ramirez-Gomez, and Jonathan Busch. Simulation of counter flow pedestrian dynamics in hallways using spheropolygons. *arXiv preprint arXiv:1310.8482*, 2013.
16. F Alonso-Marroquin. Spheropolygons: A new method to simulate conservative and dissipative interactions between 2d complex-shaped rigid bodies. *EPL (Europhysics Letters)*, 83(1):14001, 2008.
17. Fernando Alonso-Marroquín and Yucang Wang. An efficient algorithm for granular dynamics simulations with complex-shaped objects. *Granular Matter*, 11(5):317–329, 2009.
18. Dirk Helbing, Illes J Farkas, Peter Molnar, and Tamás Vicsek. Simulation of pedestrian crowds in normal and evacuation situations. *Pedestrian and evacuation dynamics*, 21:21–58, 2002.
19. T Korhonen, S Heliovaara, Simo Hostikkaa, and H Ehtamo. Counterflow model for agent-based simulation of crowd dynamics. *Safety Science*, 2010.
20. Paul A Langston, Robert Masling, and Basel N Asmar. Crowd dynamics discrete element multi-circle model. *Safety Science*, 44(5):395–417, 2006.
21. Chih-Hsien Chi, Jui-Yi Tsou, and Fong-Chin Su. Effects of rescuer position on the kinematics of cardiopulmonary resuscitation (cpr) and the force of delivered compressions. *Resuscitation*, 76(1):69–75, 2008.
22. Fernando Alonso-Marroquín, Álvaro Ramírez-Gómez, Carlos González-Montellano, Nigel Balaam, Dorian AH Hanaor, EA Flores-Johnson, Yixiang Gan, Shumiao Chen, and Luming Shen. Experimental and numerical determination of mechanical properties of polygonal wood particles and their flow analysis in silos. *Granular Matter*, 15(6):811–826, 2013.