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Further evidence on the PPP analysis of the Australian dollar. Non-linearities, fractional integration and structural change

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ABSTRACT

The aim of this paper is to analyse the empirical fulfilment of the Purchasing Power Parity (PPP) theory for the Australian dollar. In order to do so we have applied recently developed unit root tests that account for asymmetric adjustment towards the equilibrium (Kapetanios et al., 2003) and fractional integration in the context of structural changes (Robinson, 1994, and Gil-Alana, 2008). Although our results point to the rejection of the PPP hypothesis, we find that the degree of persistence of shocks to the Australian dollar decreases after the 1985 currency crisis.

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1.   Introduction

There is no doubt that Purchasing Power Parity (PPP) analysis has become one of the most controversial topics within international economics. Although the existing literature is vast, empirical contributions have yielded contradictory results, given that they are dependent on the countries, period analysed and econometric techniques employed. Although PPP is a theory of exchange rate determination, its empirical fulfilment has several policy implications; first, many macroeconomic models assume a constant real exchange rate in equilibrium, which has consequences when the policy makers base their decisions on this type of modelling; second, as Wei and Parsley (1995) claim, the PPP hypothesis can be considered as a measure of the degree of economic integration between countries, since this theory assumes perfect mobility of goods and an absence of trade barriers; finally, Faria and León-Ledesma (2003, 2005) claim that a misaligned real exchange rate (RER) might affect growth and unemployment, thus, a proper assessment of the degree of overvaluation or undervaluation of the currencies is important to help promote long term economic growth.

The PPP theory establishes that the nominal exchange rate between two currencies should be equal to the price index ratio between the countries, so that any change in the price relationship will affect the nominal exchange rate to keep the purchasing power between both countries unaffected, that is

$$E_t = \frac{P_t^*}{P_t}, \qquad\qquad (1)$$

where P_t^* is the foreign price index, P_t is the national price index, and E_t is the nominal exchange rate between both currencies.

This implies that the RER

$$Q_t = \frac{P_t E_t}{P_t^*} = 1.$$ (2)

Relations such as (1) and (2) are known as the absolute version of the PPP hypothesis. This implies that the same quantity of goods can be purchased in both countries with the same amount of money. However, this is not always true, since differences in productivity and transport costs may affect the relationship between exchange rates and prices. Then, it is possible to define a less restrictive version of the PPP hypothesis, known as the relative PPP. In this case the RER should be equal to a constant, different from 1, meaning that exchange rates react proportionally to changes in the price ratio instead of identically.

Within the empirical literature on the PPP theory it is well known that short run deviation may prevent the PPP hypothesis from holding true. However, one may expect the RER to return to its equilibrium value after a shock in the long run. Statistically this implies that the RER should be a mean reverting process for the PPP hypothesis to be fulfilled, which makes unit root testing to be an appealing econometric approach for this purpose.

Sarno and Taylor (2002), Taylor et al. (2001) and Taylor (2006), among others, provide summaries of the main contributions to the literature from which it is possible to highlight several facts; first, authors have applied different techniques since the 70s in order to capture the true data generating process (DGP) of the real exchange rates; second, it appears that the results have been quite controversial, and finally, that the most recent contributions focus on the consideration of non-linearities and fractional integration alternatives. In this vein, Taylor et al. (2001) identified two main paradoxes relating to RER behaviour. First, the relationship between nominal exchange rates and

prices implied by the PPP theory is not in many cases a cointegrating one; second, how is it possible to observe a high volatility of exchange rates in the short term with the low speed of mean reversion generally observed in this variable?

The failure of the literature to provide good answers to the aforementioned questions have been related to the low power of the (unit-root) tests applied to analyse the empirical fulfilment of PPP[1]. It appears that increasing the data sample creates additional problems, such as structural changes, that may affect the power of the tests, if these are not taken into account (see Perron and Phillips, 1987, and West, 1987, among others). Therefore, these kind of non-linearities in the deterministic components may increase the probability of Type II error with traditional unit root tests (e.g., Dickey-Fuller type). Furthermore, non-linearities in the RER long run path may be present in the form of an asymmetric speed of adjustment towards the equilibrium, i.e. the further the RER deviates from the long run equilibrium value, the faster the speed of mean reversion is expected to be. From an econometric viewpoint, that may imply an autoregressive parameter dependent on the values of the variable. The non-linear behaviour of exchange rates has been acknowledged by several authors, such as Dumas (1994), Michael at al. (1997), Sarno et al. (2004), Juvenal and Taylor (2008), and Cuestas (2009) among many others.

In addition, as pointed out by several authors such as Diebold et al. (1991), Cheung and Lai (1993) and Gil-Alana (2000) among others, the real exchange rate may be characterised as a slow mean reversion process and traditional unit root may not be

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[1] Unit root tests most commonly employed in the literature (Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al., 1992; etc.) have very low power against trend-stationarity (DeJong, Nankervis, Savin and Whiteman, 1992), structural breaks (Perron, 1989; Campbell and Perron, 1991), regime-switching (Nelson, Piger and Zivot, 2001), or fractional integration (Diebold and Rudebusch, 1991; Hassler and Wolters, 1995; Lee and Schmidt, 1996).

able to distinguish this type of process from a unit root. Thus, according to these authors, fractional integration may be an alternative viable route to the analysis of the RER dynamics. As mentioned above, the question of interest is to determine if deviations from PPP are transitory or permanent, which, translated to the fractionally integrated literature means to determine if the order of integration is smaller than 1 (mean reversion) or equal to or higher than 1 (no mean reversion). Applying R/S techniques to daily rates for the British pound, French franc and Deutsche mark, Booth et al. (1982) found evidence of fractional integration during the flexible exchange rate period (1973-1979). Cheung (1993) also found similar evidence in foreign exchange markets during the managed floating regime. On the other hand, Baum, et al. (1999) estimated ARFIMA models for real exchange rates in the post-Bretton Woods era and found almost no evidence to support long run PPP. Additional papers on exchange rate dynamics using fractional integration are Fang et al. (1994), Crato and Ray (2000) and Wang (2004).

In another contribution, Henry and Olekalns (2002) analyse the Australian RER long run behaviour in the context of structural changes and fractional integration. These authors apply Robinson's (1994) and Geweke and Porter-Hudak's (1983) fractional integration tests and the Vogelsang's (1997) unit root test with structural changes, but they are not able to find evidence of mean reversion in the RER Australian dollar. Similar results are obtained by Darné and Hoarau (2008), who applied the Perron and Rodríguez's (2003) unit root test with structural changes to the Australian dollar. Contrary to these results, Cuestas and Regis (2008) found that the Australian RER is stationary around a non-linear deterministic trend, by means of applying the Bierens (1997) unit root tests, which proposes a Chebyshev polynomial approximation for the non-linear trend. However, the results are dependent on the selection of the order of the

polynomials, since there is no unique way of doing it. Additionally, in the context of PPP it is difficult to give an economic interpretation of the non-linear trend.

The aim of the present paper is to provide further evidence of the PPP fulfilment in Australia, and complement the previous literature on the PPP analysis, by applying different techniques that address the aforementioned facts, i.e. fractional integration and non-linearities, that may affect the power of the traditional (linear) unit root tests. First, we apply the upgraded versions of linear unit root tests propossed by Ng and Perron (2001); second, we account for asymmetric speed of mean reversion and apply the Kapetanios et al. (KSS, 2003) unit root test, which generalises the alternative hypothesis to a globally stationary exponential smooth transition (ESTAR) process. Finally, in order to take into account the existence of structural changes and fractional integration at the same time, we apply Robinson's (1994) and Gil-Alana's (2008) fractional integration tests in the context of structural changes.

The remainder of this paper is organised as follows. The next section describes the data. Section 3, summarises the econometric techniques applied to empirically assess the fulfilment of PPP in Australia, and presents the results. Section 4 concludes the paper.

2.   The data

The data for this empirical analysis consists of quarterly observations of the Australian real effective exchange rate computed as a trade weighted index from 1970:2 to 2008:3, obtained from the Central Bank of Australia web page (http://www.rba.gov.au). A plot of the data is displayed in Figure 1. From this graph it is possible to highlight two main stylised facts; the existence of a structural change in the series –presumably at 1985

coinciding with the currency crisis (Darné and Hoarau, 2008)- that needs to be accounted for, and, it appears that the real value of the currency tends to revert to its equilibrium very slowly after a shock, which is suggestive of the fact that the DGP may be fractionally integrated (Henry and Olekalns, 2002).

[Insert Figure 1 about here]

3 Econometric methodology and results

3.1 Methodology

In this section we briefly describe some of the methods that will be employed in this article in order to test for the empirical fulfilment of the PPP theory.

The first tests presented are those attributable to Ng and Perron (2001), who propose several modifications to existing unit root tests in order to improve their size and power, in particular with relatively small samples. The authors present the following tests; MZ_{α} and MZ_{t} which are the modified versions of the Phillips (1987) and Phillips and Perron (1988) Z_{α} and Z_{t} tests; the MSB which is related to the Bhargava (1986) R_{1} test; and, finally, the MP_{T} test which is a modified version of the Elliot et al. (1996) Point Optimal Test. These new tests incorporate a Modified Information Criterion (MIC) to select the lag length in the auxiliary regression, given that, according to Ng and Perron (2001), the Akaike and Schwartz Information Criteria tend to select a low lag order. Additionally, these authors propose a Generalised Least Squares method of detrending the data in order to improve the power of the tests.

Secondly, as KSS point out, traditional (linear) unit root tests may fail to reject the null hypothesis when the DGP is non-linear. If the speed of adjustment is asymmetric, i.e. it actually depends on the degree of misalignment from the equilibrium, Dickey-Fuller type tests may incorrectly conclude that the series is a unit root, when in fact is a non-linear globally stationary process. In this case, we may define a DGP with two regimes, that is, an inner regime where the variable is assumed to be I(1) and an outer regime, where the variable may or may not be a unit root. The transition between regimes is smooth rather than sudden. Thus KSS propose a unit root test to analyse the order of integration of the variable in the outer regime, bearing in mind that the process is globally stationary. In other words,

$$y_t = \beta y_{t-1} + \phi y_{t-1} F(\theta; y_{t-1}) + \varepsilon_t, \qquad (3)$$

where ε_t is $iid(0, \sigma^2)$ and $F(\theta; y_{t-1})$ is the transition function, which is assumed to be exponential (ESTAR),

$$F(\theta; y_{t-1}) = 1 - exp\{-\theta y_{t-1}^2\} \qquad (4)$$

with $\theta > 0$.

In practice, it is common to reparameterise equation (3) as

$$\Delta y_t = \alpha y_{t-1} + \gamma y_{t-1}(1 - exp\{-\theta y_{t-1}^2\}) + \varepsilon_t. \qquad (5)$$

in order to apply the test. The null hypothesis $H_0 : \theta = 0$ is tested against the alternative $H_1 : \theta > 0$, i.e. we test whether the variable is an I(1) process in the outer regime. From an economic viewpoint and in the context of exchange rates, this implies that the further the RER deviates from the equilibrium, the faster will be the speed of mean reversion towards the fundamental equilibrium. In addition, the existence of trade barriers may create a central threshold where transactions are not profitable and arbitrage does not

clear the market -unit root process in the inner regime-, whereas for large deviations, the profits from arbitrage are greater than the cost, and the arbitrage mechanism brings the exchange to the inner regime. Moreover, according to Taylor and Peel (2000), among others, an ESTAR function is appropriate to model exchange rates, given that this type of equation assumes that the effects of the shock on the variable are symmetric in the sense that these effects do not depend on the sign of the shock.

On the other hand, many test statistics have been developed in recent years for fractional integration. They can be parametric or semiparametric and they can be specified in the time domain or in the frequency domain. In this article we employ two parametric approaches that allow us to incorporate structural breaks. The first one is the well-known Robinson tests (1994) that we specify in a way that permit us to include deterministic broken trends. In particular, we consider a model of form:

$$y_t \ = \ \alpha_1 \, I(t < T_b) \ + \ \alpha_2 \, I(t \geq T_b) \ + \ x_t, \tag{6}$$

$$(1 - L)^d \, x_t \ = \ u_t, \quad u_t \approx \ I(0). \tag{7}$$

where y_t is the observed time series (RER), I(x) is the indicator function, L is the lag operator ($Lx_t = x_{t-1}$), d is a real value and u_t is I(0). Based on this set-up, for a given T_b-value, we test the null hypothesis:

$$H_o : \ d \ = \ d_o, \tag{8}$$

in (6) and (7) for any real value d_o. The functional form of the test statistic is described in Appendix A and, it was shown by Robinson (1994) that, under very mild regularity conditions, a test of (8) against the one-sided alternatives: $d < d_o$ ($d > d_o$) follows the standard N(0, 1) distribution.

The method presented just above imposes the same degree of integration before and after the break-date. Then, we also perform a recent procedure developed by Gil-Alana (2008) which allows us to estimate the break-date along with the fractional differencing parameters, which might be different for each subsample. This method is based on minimising the residuals sum squares and is briefly described in Appendix B. In the final part of the article, a semiparametric method (Robinson, 1995) is conducted on the two subsamples.

3.2 Empirical results

The results of applying the Ng and Perron (2001) and KSS tests are reported in Table 1. The lag length has been selected using the Modified Akaike Information Criterion, proposed by the former authors, for both tests. In both cases the unit root hypothesis cannot be rejected at conventional significance levels. Thus, according to these (unit-root) procedures there is no evidence of the PPP hypothesis for the Australian case.

[Insert Table 1 about here]

Next, we examine the possibility of fractional integration in the context of non-linear deterministic terms. For this purpose, we employ the model given by (6) and (7), testing H_o (8) for d_o-values ranging from 0 to 2 with 0.01 increments, using the Robinson's (1994) parametric approach for fixed values of T_b. We then estimate d by choosing the value that produces the lowest statistic in absolute value, moving the break date T_b 1-period forward recursively from $T_b = 40$ (1980Q1) to $T_b = 119$ (1999Q4). Figure 2 displays the estimated d's along with the 95% confidence bands for the two

cases of white noise u_t (in Figure 2(i)) and autocorrelated disturbances (in Figure 2(ii)). In the latter case we assume that u_t follows the exponential spectral model of Bloomfield (1973). This is a non-parametric specification for the error term that produces autocorrelations decaying exponentially as in the autoregressive (AR) case.[2]

[Insert Figure 2 and Table 2 about here]

We observe in Figure 2 that the estimated values of d have remained relatively stable across T_b, with values fluctuating around 1. In fact, the unit root null hypothesis cannot be rejected for any T_b. We observe that the only turbulences in the estimators take place when T_b is around 1985. Table 2 displays the estimates of d and the intercepts for the case of a break at the four quarters in 1985, for the two cases of white noise and autocorrelated disturbances. We see in this table that practically all the estimates are slightly above 1. The only exception is the case of the break at 1985Q4 with autocorrelated disturbances where the estimate of the fractional differencing parameter is equal to 0.99. Nevertheless, the unit root cannot be rejected in any single case and thus, we do not find support for the PPP hypothesis when using this model.

Still on this set-up, we might be interested in knowing if the intercept has changed from one subsample to another. Thus, we can consider a joint test of the null hypothesis:

$$H_o : \ \alpha_1 \ = \ \alpha_2 \ \text{and} \ d \ = \ d_o, \tag{9}$$

in (6) and (7) against the alternative,

$$H_a : \ \alpha_1 \ \neq \ \alpha_2 \ \text{or} \ d \ \neq \ d_o. \tag{10}$$

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[2] See Gil-Alana (2004) for the analysis of fractional integration in the context of Bloomfield disturbances.

This possibility is not addressed in Robinson (1994) though Gil-Alana and Robinson (1997) derived a similar Lagrange Multiplier (LM) test as follows: they consider a regression model of form as:

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, ..., \tag{11}$$

and x_t given by (7) with the vector partitions $z_t = (z_{At}{}^T, z_{Bt}{}^T)^T$, $\beta = (\beta_A{}^T, \beta_B{}^T)^T$. In general, we want to test H_o: d = d_o and $\beta_B = \beta_{B0}$. Then, a LM statistic may be shown to be $\hat{r}^2$ (see Appendix A) plus

$$\sum_{t=1}^{T} \hat{u}_t \, w_{Bt}{}^T \left(\sum_{t=1}^{T} w_{Bt} w_{Bt}{}^T - \sum_{t=1}^{T} w_{Bt} w_{At}{}^T \left(\sum_{t=1}^{T} w_{At} w_{At}{}^T \right)^{-1} \sum_{t=1}^{T} w_{At} w_{Bt}{}^T \right)^{-1} \sum_{t=1}^{T} \hat{u}_t \, w_{Bt} \tag{12}$$

with $w_t = (w_{At}{}^T, w_{Bt}{}^T)^T = (1 - L)^{do} z_t,$

$$\hat{u}_t = (1-L)^{d_o} y_t - (\tilde{\beta}_A{}^T, \beta_{Bo}{}^T) w_t; \quad \tilde{\beta}_A = \left(\sum_{t=1}^{T} w_{At} \, w_{At}{}^T \right)^{-1} \sum_{t=1}^{T} w_{At} (1-L)^{d_o} y_t,$$

$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{u}_t{}^2$, and $\hat{r}^2$ is calculated as in Appendix A but using the $\hat{u}_t$ just defined. If the dimension of z_{Bt} is q_B, then we compare (12) with the upper tail of the $\chi^2_{1+q_B}$ distribution. In our case, testing (9) against (10) in (6) and (7), we have $q_B = 1$, z_{At} = I(t < T_b), z_{Bt} = I(t ≥ T_b) for t ≥ 1. Performing the test for the cases of a break at the four quarters in 1985, we obtain evidence in favour of significantly different intercepts in the case of the first two quarters and in the two cases of white noise and autocorrelated disturbances.

[Insert Figure 3 about here]

The approach employed above imposes the same degree of integration before and after the break, which might be too restrictive in some cases. In Figure 3 we examine the stability of the fractional differencing parameter across the sample period. For this purpose, we estimate d for samples of size T = 100 (i.e., 25 complete years), starting from the sample (1970Q2-1995Q2) and moving forward one period each time, ending at the sample 1983Q3-2008Q3. Again we display the estimates for the two cases of white noise and autocorrelated (Bloomfield) disturbances. Once more the estimates oscillate around the case of d = 1, and the most unstable behaviour occurs between the 12th and the 26th subsamples, including thus, the 1985 year.

Next in this section, we allow for the possibility of changes in the differencing parameter. We employ here the following model,

$$y_t = \alpha_1 + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1,...,T_b, \tag{13}$$

and

$$y_t = \alpha_2 + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1,...,T, \tag{14}$$

where α_1 and α_2 are the coefficients corresponding to the intercepts respectively for the first and second subsamples; d_1 and d_2 may be real values, u_t is I(0), and T_b is the time of a break that is supposed to be unknown. Note that given the difficulties in distinguishing between models with fractional orders of integration and those with broken deterministic trends, (Diebold and Inoue, 2001; Granger and Hyung, 2004; etc.), it is important to consider estimation procedures that deal with fractional unit roots in the presence of broken deterministic terms. We implement here the procedure developed by Gil-Alana (2008) that is based on minimising the residuals sum squared in the two subsamples. (See Appendix B).

[Insert Figure 4 and Table 3 about here]

We present the results for white noise and AR(1) disturbances.[3] In the upper plot in Figure 4 we display the RSS for the uncorrelated case and for different combinations of (T_b, d_1, d_2)-values. We observe in Table 3 that the lowest value corresponds to a break in 1985Q2, followed closely by another break at 1986Q3. In both cases, the orders of integration are slightly above 1 and the unit root cannot be rejected in any of the four subsamples. If we permit autocorrelation through the use of autoregressions, the estimated break-date occurs at exactly the same date as in the uncorrelated case, i.e., 1985Q2 and the estimated orders of integration are 0.99 for the first subsample and 0.87 for the second subsample. The second best break-date takes place at 1982Q2 and the estimated differencing parameters are 1.14 and 0.91 for the first and second subsamples respectively. Performing LR tests in the two cases with a break at 1985Q2, we obtain evidence in favour of the autocorrelated case. Thus, we observe here a decay in the degree of dependence of the series though the unit root cannot be rejected in any of the two subsamples.

To verify that there is certainly a decrease in the degree of integration of the series after the break in 1985, we also performed a semiparametric procedure for estimating d in each of the two subsamples separately. We use Robinson's (1995) Whittle approach. This method is described in Appendix C and, though there exists further refinements of this procedure (Velasco, 1999; Velasco and Robinson, 2000, Phillips and Shimotsu, 2004, 2005), these methods require additional user-chosen parameters and the estimates

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[3] Using higher AR(k) orders (k = 2, 3 and 4) lead essentially to the same results.

of d may be very sensitive to the choice of these parameters. In this respect, the use of Robinson's (1995) method seems computationally simpler. Also, this estimator is robust to a certain degree of conditional heteroskedasticity (Robinson and Henry, 1999) and is more efficient than other semi-parametric competitors. The results for the two subsamples are displayed in Figure 5.

[Insert Figure 5 about here]

Throughout the estimates we also present in Figure 5 the 95% confidence band corresponding to the I(1) hypothesis. We report the estimates for the whole range of values of the bandwidth number m (m = 1, 2, …, T/2)[5], and though some attempts have been made to determine the optimal bandwidth number in semiparametric long memory models (Robinson and Henry, 1996), in the context of the Whittle estimate employed here, it has not yet been theoretically justified. We see that for the first subsample, practically all the estimates are within the I(1) interval, though if the bandwidth number is large, the values of d are very close to the upper band, being even significantly above 1 in some cases. In the case of the second subsample, the estimates are also within the I(1) confidence band though they are generally smaller than in the previous case.

These results have an important implication for the exchange rate policy of Australia; the fact that the degree of shock persistence of to the RER has decreased after the 1985's currency crisis, provides us with an insight into the degree of success of the policies applied to overcome the crisis and keep the value of the currency more stable. In

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[5] The choice of the bandwidth number in this method is crucial since it balances the trade-off between bias and variance. The asymptotic variance of this estimator is decreasing with m while the bias is growing with m.

addition, caution is needed when applying macroeconomic models that assume constant real exchange rates, for the Australian case. For this country, and giving its external dependence on commodities, the RER may be explained by the commodities terms of trade as claimed by Casin et al. (2004).

4.   **Conclusions**

Aiming at contributing to the empirical literature on the PPP fulfilment for the Australian dollar, we have applied some recently developed unit root tests, which take into account the possibilities of asymmetric speed of adjustment towards the equilibrium, as well as fractional integration and structural changes. Our results are in line with previous empirical works, in the sense that there is no evidence of mean reversion in the Australian Dollar RER, for the period analysed. However, we find that the order of integration decreases after 1985, coinciding with the currency crisis. This conclusion highlights the fact that the monetary authorities have managed to decrease the dependence of the Australian Dollar on real shocks that may affect the value of the currency on a permanent basis.

Appendix A

The LM test of Robinson (1994) for testing H$_o$ (8) in (6) and (7) is

$$\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{-1/2} \hat{a},$$

where T is the sample size and:

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} \;=\; \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 \;-\; \sum_{j=1}^{T-1}\psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\varepsilon}(\lambda_j) \;=\; \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \qquad \lambda_j \;=\; \frac{2\pi j}{T}; \qquad \hat{\tau} \;=\; \arg\min \sigma^2(\tau).$$

$\hat{a}$ and $\hat{A}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to d (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of u_t evaluated under the null, i.e.:

$$\hat{u}_t \;=\; (1-L)^{d_o}\,y_t \;-\; \hat{\alpha}'w_t;$$

$$\hat{\alpha} \;=\; \left(\sum_{t=1}^{T}w_t\,w_t'\right)^{-1}\sum_{t=1}^{T}w_t\,(1-L)^{d_o}\,y_t; \qquad w_t \;=\; (1-L)^{d_o}\,z_t; \quad z_t \;=\; (1I(t<T_b),\, 1I(t\geq T_b))',$$

and g is a known function related to the spectral density function of u_t:

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi}\,g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi.$$

Appendix B

Gil-Alana (2008) considers the following model,

$$y_t = \alpha_1 + x_t; \quad (1-L)^{d_1}x_t = u_t, \quad t = 1,...,T_b,$$

and

$$y_t = \alpha_2 + x_t; \quad (1-L)^{d_2}x_t = u_t, \quad t = T_b+1,...,T,$$

where the α_1 and α_2 are the intercepts corresponding to the first and second subsamples respectively; d_1 and d_2 may be real values, u_t is I(0), and T_b is the time of a break that is supposed to be unknown. This model can be re-parameterized as follows:

$$(1-L)^{d_1}y_t = \alpha_1\,\tilde{1}_t(d_1) + \beta_1\tilde{t}_t(d_1) + u_t \qquad\qquad t=1,...,T_b$$

$$(1-L)^{d_2} y_t = \alpha_2 \tilde{1}_t(d_2) + \beta_2 \tilde{t}_t(d_2) + u_t \qquad\qquad t = T_b + 1,...,T$$

with $\tilde{1}_t(d_i) = (1-L)^{d_i} 1$. The idea is, then, to minimise the residual sum of squares (RSS),

$$\min_{w.r.t\{\alpha_1,\alpha_2,\beta_1,\beta_2\}} \sum_{t=1}^{T_b} \left[(1-L)^{d_{1o}^{(1)}} y_t - \alpha_1 \tilde{1}_t(d_{1o}^{(1)}) + \beta_1 \tilde{t}_t(d_{1o}^{(1)})\right]^2 + \sum_{t=T_b}^{T} \left[(1-L)^{d_{2o}^{(1)}} y_t - \alpha_2 \tilde{1}_t(d_{2o}^{(1)}) + \beta_2 \tilde{t}_t(d_{2o}^{(1)})\right]^2$$

In so doing, it is necessary to choose a grid for the values of the differencing parameters d_1 and d_2, d_{1o}, and d_{2o} say. Once the estimated parameters, $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1$ and , $\hat{\beta}_2$ are obtained for partition T_b and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$, we plug these values into the objective function and obtain $RSS(T_b) = \arg\min_{\{i,j\}} RSS(T_b; d_{1o}^{(i)}, d_{1o}^{(j)})$. In order to estimate the time break, $\hat{T}_k$, we obtain the moment that minimises the RSS, where the minimisation is taken over all partitions $T_1, T_2,...,T_m$, such that $T_i - T_{i-1} \geq |\varepsilon T|$. Then, it is possible to obtain the regression parameter estimates as $\hat{\alpha}_i = \hat{\alpha}_i(\hat{T}_k)$, and the differencing parameters, $\hat{d}_i = \hat{d}_i(\hat{T}_k)$, for $i=1,2$.

Appendix C

The "local" Whittle estimate of Robinson (1995) is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right),$$

$$\text{for } d \in (-1/2, 1/2); \quad \overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0.$$

where m is a bandwidth parameter number and d $\in$ (-0.5, 0.5). Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroskedasticity (Robinson and Henry, 1999) and is more efficient than other semi-parametric competitors.

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Table 1: Ng-Perron and KSS unit root test results

Test	Statistic	CV (5%)	CV (10%)
MZ_{α}	-2.22737	-8.10000	-5.70000
MZ_t	-1.05215	-1.98000	-1.62000
MSB	0.47237	0.23300	0.27500
MP_t	10.9753	3.17000	4.45000
$\hat{t}_{NLD}$	-1.95642	-2.91689	-2.63526

Note: The order of lag to compute the tests has been chosen using the modified AIC (MAIC) suggested by Ng and Perron (2001). The Ng-Perron tests include an intercept, whereas the KSS test has been applied to the demeaned data, $\hat{t}_{NLD}$ say. The critical values for the Ng-Perron tests have been taken from Ng and Perron (2001), whereas those for the KSS have been obtained by Monte Carlo simulations with 50,000 replications.

Table 2: Estimates of intercepts and fractional differencing parameters with a break in 1985

Break Dates	White noise disturbances			Autocorrelated disturbances		
	d	α_1	α_2	d	α_1	α_2
1985Q1	1.05 (0.95, 1.17)	1.500 (1.497, 1.503)	1.407 (1.400, 1.413)	1.03 (0.85, 1.29)	1.500 (1.497, 1.503)	1.404 (1.398, 1.410)
1985Q2	1.04 (0.95, 1.17)	1.500 (1.497, 1.503)	1.354 (1.348, 1.360)	1.03 (0.84, 1.27)	1.500 (1.497, 1.503)	1.353 (1.347, 1.359)
1985Q3	1.10 (0.99, 1.25)	1.501 (1.498, 1.504)	1.556 (1.549, 1.562)	1.06 (0.88, 1.32)	1.500 (1.497, 1.504)	1.546 (1.539, 1.552)
1985Q4	1.06 (0.97, 1.19)	1.500 (1.497, 1.504)	1.435 (1.429, 1.442)	0.99 (0.82, 1.22)	1.499 (1.496, 1.503)	1.428 (1.422, 1.435)

Note: In parenthesis, the 95% confidence intervals.

Table 3: Estimates of the parameters based on the procedure

	Break date	First sub-sample			Second sub-sample		
		d_1	α_1	ρ_1	d_2	α_2	ρ_2
		i) White noise disturbances					
[1]	1985Q2	1.07 (0.88, 1.25)	1.501 (30.050)	---	1.05 (0.92, 1.25)	1.143 (28.997)	---
[2]	1986Q3	1.04 (0.90, 1.25)	1.500 (28.860)	---	1.09 (0.96, 1.20)	0.965 (26.745)	---
		ii) AR(1) disturbances					
[3]	**1985Q2**	**0.99 (0.87, 1.12)**	**1.499 (29.684)**	**-0.116**	**0.87 (0.73, 1.04)**	**1.131 (28.975)**	**0.202**
[4]	1982Q2	1.14 (0.88, 1.57)	1.502 (32.016)	-0.130	0.91 (0.78, 1.22)	1.344 (31.722)	0.212

Note: In parenthesis, the 95% confidence intervals for the values of the d's, and the t-values in case of the intercepts.
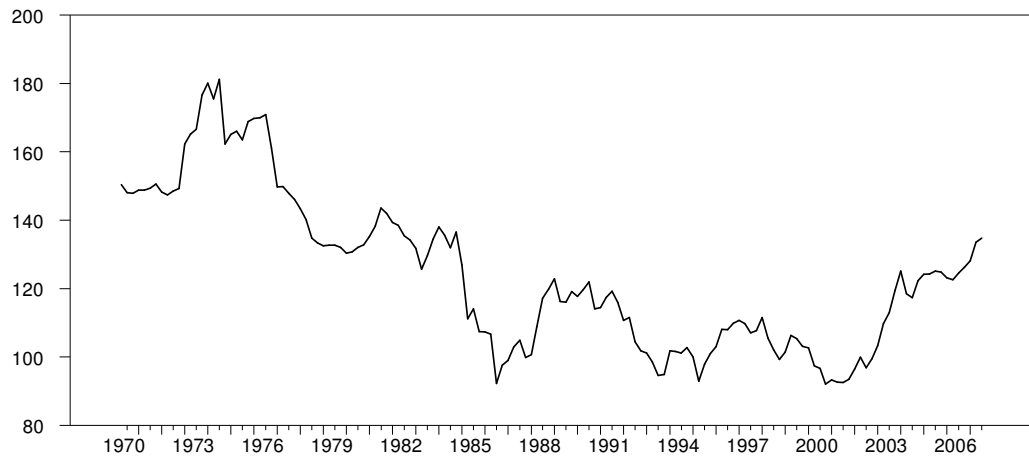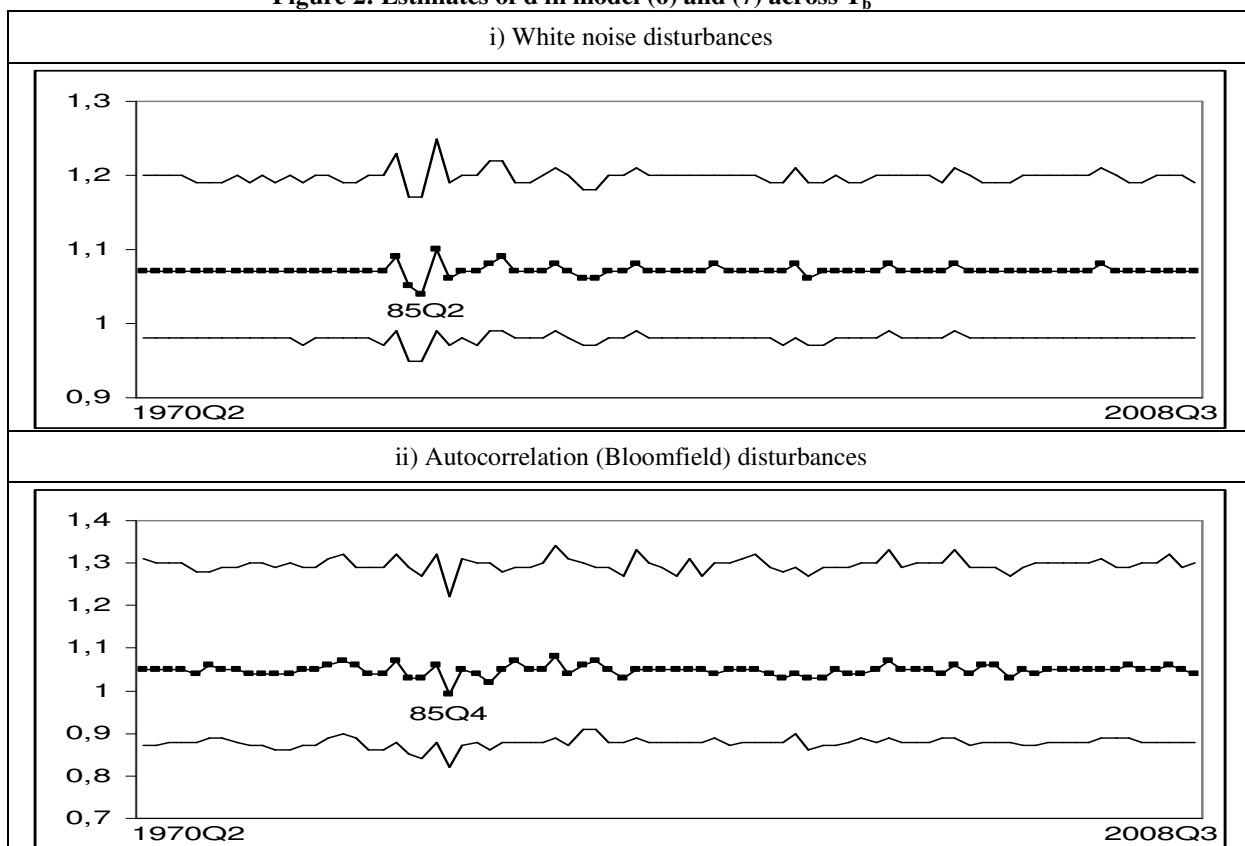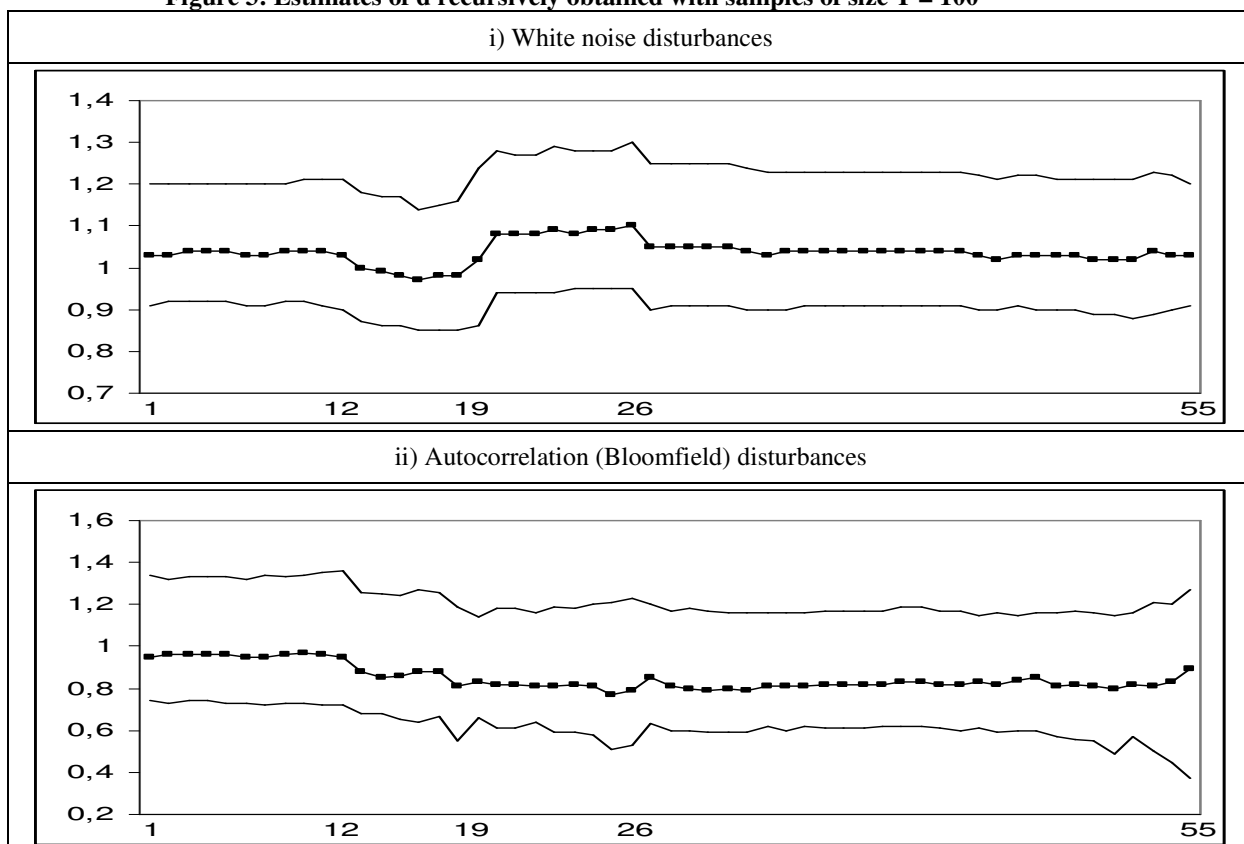
Figure 1: **Australian RER**

Figure 2: Estimates of d in model (6) and (7) across T_b

Note: The thick line refers to the estimated values of d, while the thin ones refer to the 95% confidence band.

Figure 3: Estimates of d recursively obtained with samples of size T = 100

i) White noise disturbances

ii) Autocorrelation (Bloomfield) disturbances

Note: The thick line refers to the estimated values of d, while the thin ones refer to the 95% confidence band.

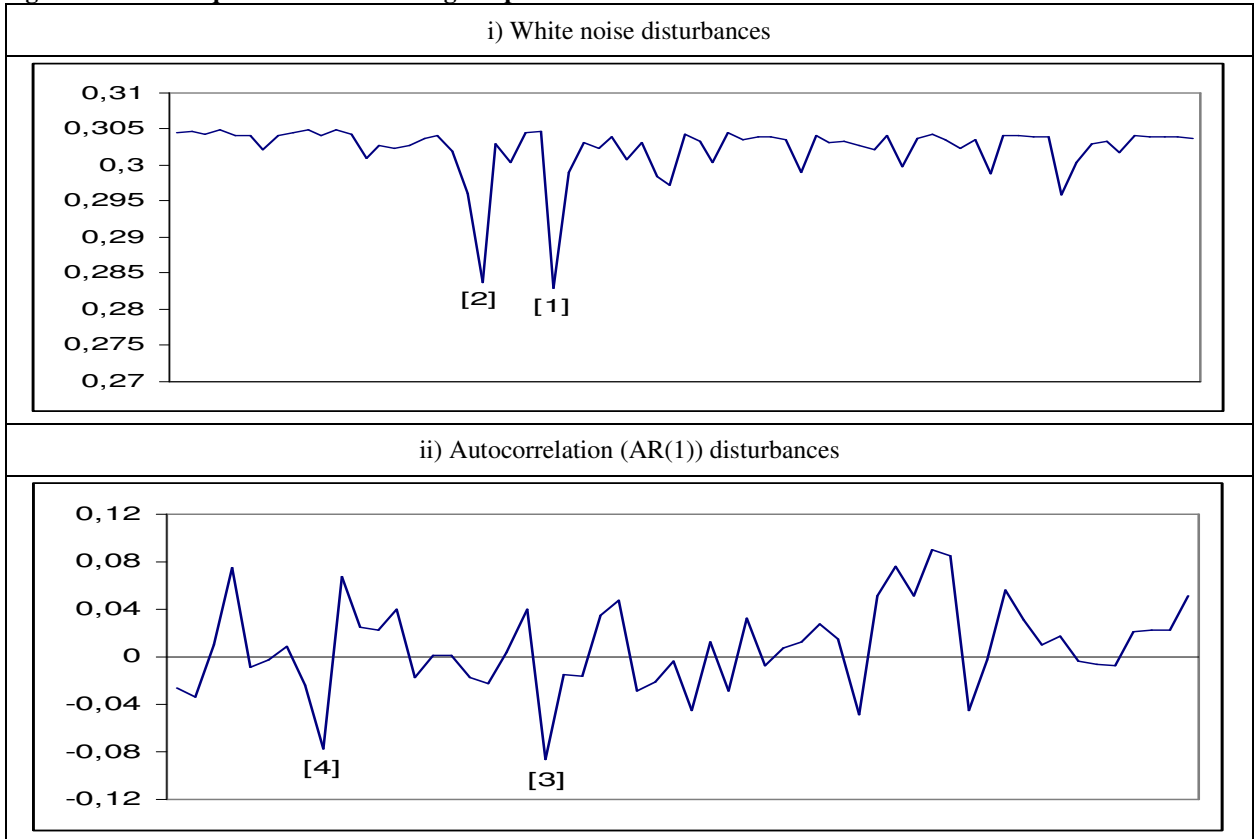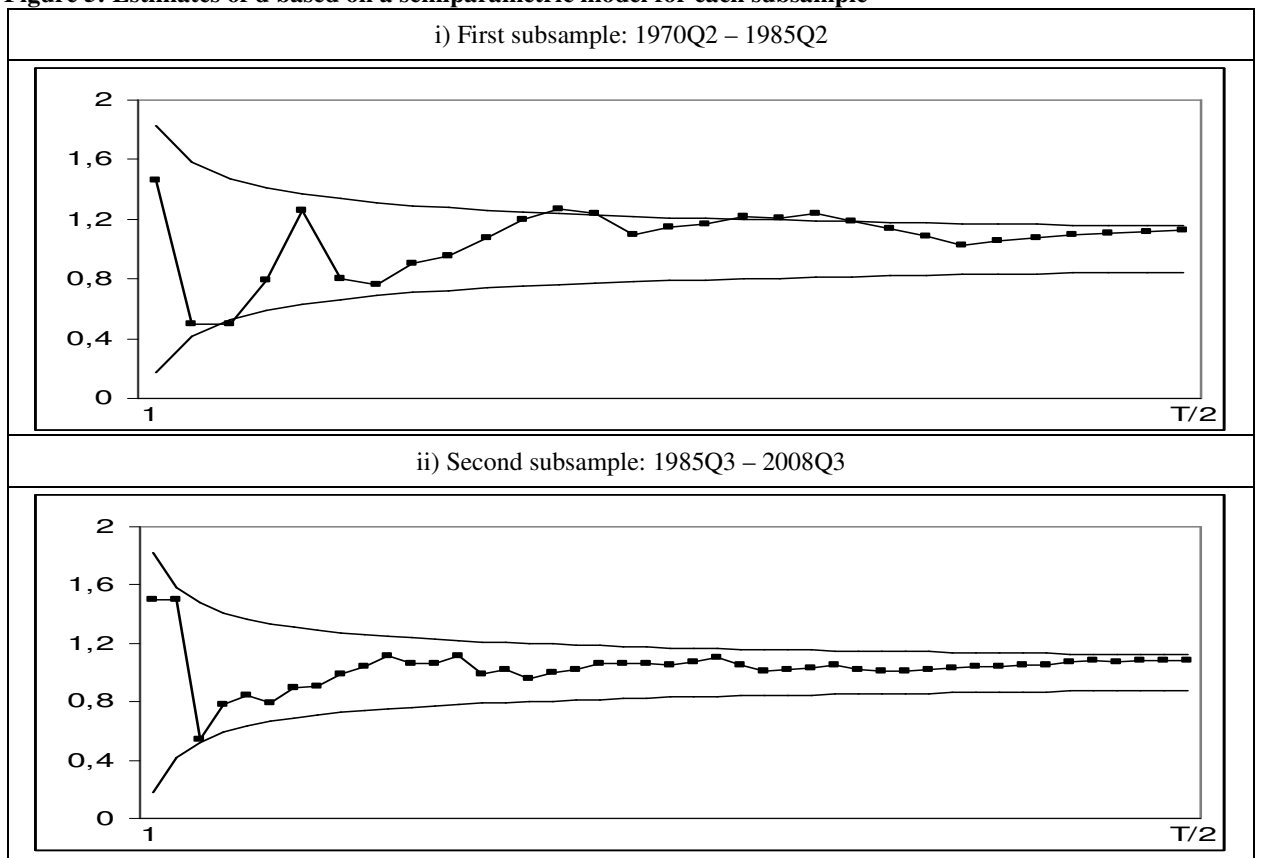**Figure 4: Sum of squared residuals using the procedure of Gil-Alana**

i) White noise disturbances

ii) Autocorrelation (AR(1)) disturbances

Figure 5: Estimates of d based on a semiparametric model for each subsample

i) First subsample: 1970Q2 – 1985Q2

ii) Second subsample: 1985Q3 – 2008Q3

Note: The horizontal axe refers to the bandwidth parameter number m, while the vertical one displays the estimates of d.