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Structural Change and the Order of Integration in Univariate Time Series

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ABSTRACT

In this article I investigate whether the presence of structural breaks affects inference on the order of integration in univariate time series. For this purpose, we make use of a version of the tests of Robinson (1994) which allows us to test unit and fractional roots in the presence of deterministic changes. Several Monte Carlo experiments conducted across the paper show that the tests perform relatively well in the presence of both mean and slope breaks. The tests are applied to annual data on German real GDP, the results showing that the series may be well described in terms of a fractional model with a structural slope break due to World War II.

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1. Introduction

It is a well-known stylised fact that many macroeconomic and financial time series contain fractional roots. Examples in the literature are Diebold and Rudebush (1989), Baillie and Bollerslev (1994), Gil-Alana and Robinson (1997), etc. However, the implication of structural change on fractionally integrated models is something that has been little investigated in economics. The possibility of confusing long memory and structural change was examined in a number of papers, including applied hydrology (e.g., Klemes, 1974); econometrics (Hidalgo and Robinson, 1996; Lobato and Savin, 1997); and mathematical statistics (eg, Künsch, 1986, Teverovsky and Taqqu, 1997), but they had little impact. More recently, Diebold and Inoue (1999) provide both theoretical and Monte Carlo evidence that structural breaks-based models and long memory processes are easily confused. Similarly, Granger and Hyung (1999) also developed a theory relating both types of models and Gil-Alana (2001a) shows that the order of integration of some series may be reduced by the inclusion of dummy variables for the breaks.

In this article, we want to investigate if the presence of structural breaks affects to the order of integration in univariate time series. For this purpose, we use a version of the tests of Robinson (1994) which is described in Section 2. Section 3 contains several Monte Carlo experiments studying the size and the power properties of the tests in the context of structural breaks. An empirical application, using annual data for the real GDP in Germany, is carried out in Section 4 and a new statistic is also developed in this section to examine the importance of the break in this series. Section 5 concludes.

2. The tests of Robinson (1994)

Following Bhargava (1986), Schmidt and Phillips (1992) and others on parameterization of unit root models, we can consider the regression model,

$$y_t \; = \; \beta' z_t \; + \; x_t, \qquad t \; = \; 1, 2, ... \tag{1}$$

where y_t is the time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of deterministic regressors; and x_t is such that

$$(1 - L)^d \; x_t \; = \; u_t, \qquad t \; = \; 1, 2, ..., \tag{2}$$

where u_t is an I(0) process, defined as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d \; = \; d_o, \tag{3}$$

in (1) and (2) for any given real value d_o. Specifically, the test statistic is then given by:

$$\hat{r} \; = \; \left(\frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{4}$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) \, g(\lambda_j; \hat{\tau})^{-1} \, I(\lambda_j); \qquad \hat{\sigma}^2 \; = \; \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} \, I(\lambda_j);$$

$$\hat{A} \; = \; \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 \; - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \; \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \quad \hat{\varepsilon}(\lambda_j) \; = \; \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j \; = \; \frac{2\pi \, j}{T}; \quad \hat{\tau} \; = \arg\min_{\tau \in \mathrm{T}} \sigma^2(\tau).$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$:

$$\hat{u}_t \; = \; (1 \; - \; L)^{d_o} \, y_t \; - \; \hat{\beta} \, w_t, \qquad w_t \; = (1 - L)^{d_o} \, z_t; \qquad \hat{\beta} \; = \left(\sum_{t=1}^{T} w_t \, w_t' \right)^{-1} \sum_{t=1}^{T} w_t \, (1 - L)^{d_o} \, y_t.,$$

and the function g above is a known function coming from the spectral density function of u_t. Thus, for example, if u_t is white noise, g ≡ 1 and, if u_t is an AR process of form: $\phi_p(L) \, u_t = \varepsilon_t$, $g = |\phi_p(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are function of τ.

Based on (3), Robinson (1994) showed that under certain regularity conditions,

$$\hat{r} \rightarrow_d N(0,1) \quad as \quad T \rightarrow \infty, \tag{5}$$

and also the Pitman efficiency property of the tests against local departures from the null.[1] Thus, we are in a classical large-sample testing situation by reasons described in Robinson (1994). A test of (3) will reject H_o against H_a: $d > d_o$ if $\hat{r} > z_\alpha$, where the probability that a standard normal variate exceeds z_α is α. Conversely, we will reject H_o (3) against H_a: $d < d_o$ if $\hat{r} < -z_\alpha$. We should finally remark here that this standard asymptotic distribution holds across the different values of d_o and also across the different regressors in z_t. Thus, we may include dummy variables to describe the structural breaks with no effect on the limit distribution of the tests.

3. A Monte Carlo experiment

We investigate in this section the rejection frequencies of the tests of Robinson (1994) against fractional alternatives in the context of structural breaks. Initially, we assume that the true model is given by (1) and (2) with $z_t = S_t$ and $S_t = S_{1t} = I(t > T/2)$, (in Table 1), and $S_t = S_{2t} = (t - T/2)I(t > T/2)$, (in Table 2); $\beta = d = 1$, i.e., we assume that the true model contains a unit root and a structural (mean and slope) break in the middle of the sample. We examine the tests for H_o: $d = d_o$, where d_o = 0.50 (0.10), 1.50 and the test statistic ($\hat{r}$ in (4)) is calculated first with $\beta = 0$ in (1), i.e., assuming that there is no break, and then with β unknown. We have generated Gaussian series, using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986) with 10,000 replications in each case. The sample sizes are 100, 200 and 300 and the nominal size is 5% in all cases.

Table 1 reports the rejection frequencies of $\hat{r}$ in (4) against both types of alternatives ($d < d_o$ and $d > d_o$) in the context of a mean shift break. Starting with the sizes of the tests,

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[1] The Pitman property refers to the fact that if we direct the tests against local alternatives: H_a: $d = d_o + \delta T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal, with variance 1 and mean that cannot be exceeded by any rival regural statistic.

we observe that they are too small when directed against H_a: d > 1, but too large against H_a: d < 1, however they considerably improve as we increase the number of observations. Thus, if T = 100, the sizes against d > 1 and d < 1 are respectively 3.1% and 10.4% with T = 100. They improve to 3.6% and 8.3% with T = 200, and if T = 300, they become 4.2% and 6.5%. We also observe that if we do not take into account the break, the rejection probabilities are relatively high when testing with values of d smaller than or equal to 0.8 or greater than or equal to 1.2, especially if the sample size is large. Testing, for example, H_o (3) with d_o = 0.8 and 1.2, the rejection frequencies are 0.914 and 0.960 with T = 200, and they become 0.987 and 0.997 if T = 300. However, if we test for orders of integration close to one, these probabilities are generally low, suggesting that the tests can properly detect the order of integration independently of the existence of a structural break. If we now perform the tests including a potential mean break, the rejection probabilities are now slightly smaller than in the previous case though we see that if T = 300 and d ≤ 0.8 or d ≥ 1.2, they are practically 1 in all cases.

(Tables 1 and 2 about here)

In Table 2 we perform the same experiment but the true model consists now of an I(1) process with a slope break. Similarly to Table 1, there is a bias in the sizes, being too small against H_a: d > 1 but too large against H_a: d < 1, however, if T = 300, the values seem to be close to the nominal size of 5%. We observe here that if we do not include the break in the regression model (1), the rejection frequencies when testing with d ≤ d_o are practically 1, however, testing with d ≥ d_o, these probabilities are very low if d = 1 (against d > 1) or d = 1.1 or 1.2. Including a slope break, the tests substantially improve, and the rejection frequencies are higher than 0.9 for all cases if d ≤ 0.8 or d ≥ 1.2.

In the previous experiments, we have only considered the cases of simple breaks in the mean and the slope. (In fact, in the second example, we assume there is no break in the first half of the sample and a trend in the second). Next, we consider a more general case of genuine breaks in a linear time trend, and assume that the true model is given by (1) and (2) with $z_t = [t, S_{1t}, S_{2t}]'$; $\beta = (1, 1, 1)'$ and d = 1, and test the same null hypothesis as in Tables 1 and 2, i.e, d_o = 0,5, (0.1), 1.5. The rejection frequencies are given in Table 3.

(Table 3 about here)

As expected, the inclusion of such deterministic components affects to the size of the tests. Thus, if T = 100, there is a clear bias in the size and the rejection probabilities are 1.3% against d > 1 and 14.1% adjacent d < 1. However, and similarly to the previous cases, they improve with T: if T = 200, the sizes are 1.7% and 10.2%, and if T = 300 they become respectively 2.3% and 9.6%, still observing a bias in the size. If we concentrate on the rejection frequencies, they are fairly good, and if the sample size is large enough, they always exceed 0.9 for d ≤ 0.8 and d ≥ 1.2. One way of sorting out this size problem might be to produce finite-sample critical values via either simulations or bootstrap. However, these finite sample critical values would be affected by the deterministic trends and thus, we would obtain different critical values depending on the nature and the time of the breaks. In that respect, and given that the rejection frequencies were relatively high in practically all cases, we have preferred in the empirical application below, to work with the asymptotic critical values given by the normal distribution.

Similar experiments were also carried out based on AR(1) and AR(2) disturbances and, though we do not reproduce the results in the paper, they were fairly similar to those displayed here for the case of white noise u_t when the roots are relatively away from 1. However, if these roots are close to unity, the performance of the tests was poor, something that is common to practically all unit-root tests existing in the literature.

4. An empirical application

The time series analysed in this section are annual data of the German real GDP for the time period 1870 – 2000. A plot of this series is given in Figure 1, observing a clear slope break in 1946 due to the World War II. Denoting the time series y_t, we employ throughout model (1) and (2), with $z_t = (1, S_t)'$, i.e.,

$$y_t = \alpha + \beta S_t + x_t, \qquad t = 1, 2, ... \tag{6}$$

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{7}$$

and $S_t = S_{2t} = (t - T^*) I(t > T^*)$ with T^* corresponding to the time period 1946. We test H_o (3) for values d_o equal to 0.50 (0.10), 1.50, and $\alpha = \beta = 0$ a priori, (i.e., including no regressors in the undifferenced regression); α unknown and $\beta = 0$ a priori, (i.e., with an intercept); $\alpha = 0$ a priori and β unknown, (i.e., with a structural break); and finally α and β unknown, (i.e., with an intercept and a slope break). Initially, we assume that the disturbances u_t are white noise (in Table 4), though later we also allow for weakly parametrically autocorrelated disturbances (in Tables 5 and 6). Other forms of structural breaks, like mean shifts ($S_t = S_{1t}$) or general breaks in the time trend ($S_t = [S_{1t}, S_{2t}]'$) were also tried. In the first case, the coefficients were insignificantly different from 0 in all case while in the latter model, only the coefficients for S_{2t} were significant in some cases, suggesting that (6) and (7) might be an adequate model specification for this series.[2]

(Figure 1 and Table 4 about here)

The test statistic reported across Table 4 (and also in Tables 5 and 6) is the one-sided one given by $\hat{r}$ in (4). Thus, significantly positive values of this are consistent with higher orders of integration whereas significantly negative ones are consistent with smaller values of d_o. A notable feature observed across Table 4 (in which u_t is white noise) is the fact that $\hat{r}$ monotonically decreases with d_o. This is something to be expected in view of the previous discussion since it is a one-sided statistic. Thus, for example, if H_o (3) is rejected with $d_o = 1$ against the alternative $d > 1$, an even more significant result in this direction should be expected when $d_o = 0.75$ or $d_o = 0.50$ are tested. We see in this table that the unit root null hypothesis (i.e., $d_o = 1$) is rejected for all type of regressors in favour of alternatives with higher orders of integration. Thus, if we do not include regressors or only an intercept is included in the regression model (6), H_o (3) cannot be rejected when $d_o = 1.3$, 1.4 and 1.5. However, if we allow for a structural break (with or without an intercept), the values of d where H_o (3) cannot be rejected are slightly smaller, ranging between 1.1 and 1.4. Thus, the results in this table are consistent with those obtained in Gil-Alana (2001a), who showed that the order of integration of some series might be reduced when deterministic changes are included in the regression model.

(Tables 5 and 6 about here)

However, the significance of the above results may be in large part due to the un-accounted for I(0) autocorrelation in u_t. Thus, in Tables 5 and 6, we allow respectively for AR(1) and AR(2) disturbances. Higher order autoregressive processes were also considered and the results, though not reported here, were very similar to those given in these two tables.[3] Starting with the case of AR(1) disturbances, we observe that if we do not include regressors or only include an intercept, there is a lack of monotonic decrease in the value of the test statistic with respect to d_o for small values of d_o. This may be an indication of model misspecification (as is argued in Gil-Alana and Robinson, 1997). Note that in the event of misppecification, monotonicity is not necessarily to be expected: frequently, misspecification inflates both numerator and denominator of $\hat{r}$, to varying degrees, and thus affects $\hat{r}$ in a complicated way. However, we also observe in this table that if the slope

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[2] Note that these coefficients are all based on the differenced model, which is short memory under the null and thus, standard tests apply.

[3] In fact, imposing AR(3) disturbances, the third AR coefficient was insignificantly different from 0, and the non-rejection values of d took place at exactly the same (z_t/d) combination as in the AR(2) u_t case.

break is included in (6), monotonicity is achieved, suggesting that this variable may be required when modelling this series. The non-rejection values of d range now between 0.6 and 1.1 and the lowest statistics are obtained when d is around 0.7. If u_t is AR(2), (Table 6), the results are very similar. There is a lack of monotonicity in the values of $\hat{r}$ with respect to d_o if we do not include the break, and this is sorted out when the dummy variable is included in the regression model, H_o (3) being then non-rejected when d is between 0.6 and 1.2. Similarly to Table 5, the lowest statistics are obtained here when d is around 0.7, implying that the series is nonstationary but with a mean reverting behaviour.

Finally, in order to deeper examine the importance of the structural break, we have considered a joint test of the null hypothesis:

$$H_o: \ d \ = \ d_o \quad and \quad \beta \ = \ 0, \tag{8}$$

against the alternative:

$$H_o: \ d \ \neq \ d_o \quad or \quad \beta \ \neq \ 0, \tag{9}$$

in (6) and (7). A joint statistic of this hypothesis was proposed in Gil-Alana and Robinson (1997) and is given by:

$$\hat{r}^2 \ + \ \frac{\left(\sum_{t=1}^{T} \widetilde{u}_t \, w_{2t} \right)^2}{\left[\sum_{t=1}^{T} w_{2t}^2 \ - \ \left(\sum_{t=1}^{T} w_{1t} \, w_{2t} \right)^2 \Big/ \sum_{t=1}^{T} w_{1t}^2 \right]} \tag{10}$$

$$w_{1t} \ = (1 - L)^{d_o} \, 1; \quad w_{2t} \ = (1 - L)^{d_o} \, S_t; \quad \widetilde{u}_t \ = (1 - L)^{d_o} \, y_t \ - \ \widetilde{\beta}_1 \, w_{1t};$$

$\widetilde{\beta}_1 = \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \sum_{t=1}^{T} w_{1t} \, (1 - L)^d \, y_t; \quad \hat{\sigma}^2 \ = \frac{1}{T} \sum_{t=1}^{T} \widetilde{u}_t^{\ 2},$ and $\hat{r}^2$ calculated as in (4) but using the $\widetilde{u}_t$ just defined. We can compare (10) with the upper tail of the χ_2^2 distribution. The results are given in Table 7.

(Table 7 about here)

We observe that the null hypothesis (8) is rejected in this table for all values of d_o and all types of disturbances, giving thus further support for the need of a slope break when modelling this series.

5. Concluding comments

Annual data of the German real GDP have been examined in this article by means of using fractionally integrated techniques. We have used of a version of the tests of Robinson (1994) that permits us to test unit and fractional roots in the presence of deterministic changes. Several Monte Carlo experiments were conducted across the paper in order to examine the power properties of the tests in the context of structural breaks, the results showing that the tests perform relatively well in the presence of both, mean and slope breaks. The tests were then applied to annual data of German real GDP and the results show that if the disturbances are white noise, the orders of integration are smaller when the structural break is considered. Allowing weakly autocorrelated disturbances, we observe a lack of monotonic decrease in the value of the test statistic with respect to the order of integration if we do not include the break, implying that the slope break due to the World War II is required when modelling this series. This is reinforced when a joint test for simultaneously testing the degree of integration and the need for the break is performed. We can therefore conclude the analysis of this series by saying that the real GDP in Germany may be well described in terms of a fractionally integrated I(d) model with d smaller than one and with a slope break in 1946. Other possible breaks (like one due to World War I, 1941-18) were also examined but was not found any evidence of significance across the sample.

This article can be extended in several directions. First, the date of the break can be taken as unknown and thus, it might be considered endogenous as in Christiano (1992), Zivot and Andrews (1992) and Banerjee et al. (1992), in case of testing for unit roots. Also, there exist other versions of the tests of Robinson (1994), based on seasonal (quarterly and monthly) and cyclical data (see, e.g. Gil-Alana and Robinson, 2001, and Gil-Alana, 1999, 2001b), and the tests can clearly be extended to allow breaks at known or unknown periods of time. Work in these directions is now under progress.

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FIGURE 1

Real Gross Domestic Product in Germany

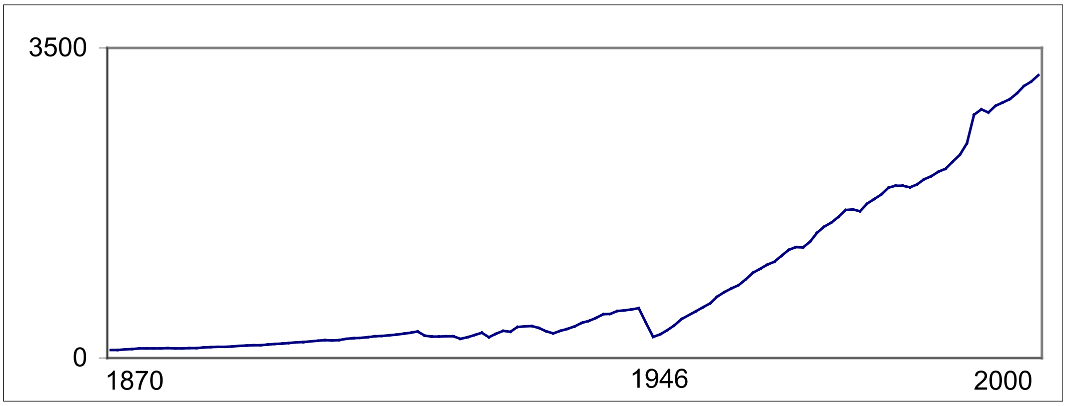

TABLE 1

Rejection frequencies of the tests of Robinson (1994) in the context of structural breaks

True model: $y_t = S_{1tt} + x_t; \quad (1 - L)\,x_t = u_t; \quad S_{1t} = I\,(t > T/2).$

: $y_t = \beta'\,S_{1t} + x_t; \quad (1 - L)^d\,x_t = u_t.$

T = 100	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	0.998	0.984	0.898	0.635	0.242	0.029	0.102	0.390	0.776	0.959	0.996	0.999
Struct. break	0.997	0.978	0.880	0.609	0.232	**0.031**	**0.104**	0.379	0.765	0.955	0.995	0.999
T = 200	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	0.999	0.995	0.914	0.454	0.037	0.084	0.567	0.960	0.999	1.000	1.000
Struct. break	1.000	0.999	0.994	0.905	0.443	**0.036**	**0.083**	0.560	0.957	0.999	1.000	1.000
T = 300	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	0.987	0.621	0.035	0.078	0.694	0.997	1.000	1.000	1.000
Struct. break	1.000	1.000	1.000	0.985	0.614	**0.042**	**0.065**	0.685	0.997	1.000	1.000	1.000

In bold, the sizes of the tests. The nominal size is 0.050.

TABLE 2

Rejection frequencies of the tests of Robinson (1994) in the context of structural breaks

True model: $y_t = S_{2t} + x_t; \quad (1 - L)\,x_t = u_t; \quad S_{2t} = (t - T/2)\,I\,(t > T/2).$

$y_t = \beta'\,S_{2t} + x_t; \quad (1 - L)^d\,x_t = u_t.$

T = 100	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	1.000	0.998	0.910	0.001	0.007	0.172	0.724	0.973	0.998
Struct. break	0.997	0.981	0.886	0.589	0.193	**0.021**	**0.120**	0.415	0.786	0.957	0.995	0.999
T = 200	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	1.000	1.000	0.998	0.000	0.008	0.257	0.965	1.000	1.000
Struct. break	1.000	1.000	0.995	0.904	0.406	**0.027**	**0.0.95**	0.591	0.963	0.999	1.000	1.000
T = 300	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.356	0.999	1.000	1.000
Struct. break	1.000	1.000	1.000	0.980	0.597	**0.036**	**0.076**	0.723	0.997	1.000	1.000	1.000

In bold, the sizes of the tests. The nominal size is 0.050.

TABLE 3												
Rejection frequencies of the tests of Robinson (1994) in the context of structural breaks												
True model: $y_t = t + S_{1t} + S_{2t} + x_t; \quad (1 - L) x_t = u_t;$												
$y_t = \beta_0 t + \beta_1 S_{1t} + \beta_2 S_{2t} x_t; \quad (1 - L)^d x_t = u_t.$												
T = 100	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	0.998	0.993	0.815	0.000	0.008	0.189	0.808	0.996	1.000
Struct. break	0.996	0.903	0.831	0.413	0.107	**0.013**	**0.141**	0.465	0.843	0.999	1.000	1.000
T = 200	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	1.000	0.999	0.876	0.000	0.008	0.231	0.889	1.000	1.000
Struct. break	0.999	0.998	0.839	0.764	0.234	**0.017**	**0.102**	0.600	0.999	1.000	1.000	1.000
T = 300	H_a: d > d_o						H_a: d < d_o					
	0.5	0.6	0.7	0.8	0.9	1	1	1.1	1.2	1.3	1.4	1.5
No break	1.000	1.000	1.000	1.000	1.000	0.994	0.000	0.022	0.241	0.904	1.000	1.000
Struct. break	1.000	0.999	0.900	0.880	0.334	**0.023**	**0.096**	0.604	0.999	1.000	1.000	1.000

In bold, the sizes of the tests. The nominal size is 0.050.

TABLE 4										
Testing H_o (3) in (1) and (2) with white noise u_t										
z_t / d	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\alpha = \beta = 0$	24.24	23.10	20.93	17.49	13.04	8.50	4.74	2.07	**0.31'**	**-0.84'**	**-1.64'**
α unknown $\beta = 0$	21.41	20.91	19.66	16.88	12.78	8.39	4.70	2.06	**0.33'**	**-0.80'**	**1.59'**
$\alpha = 0$ β unknown	11.57	9.24	7.15	5.30	3.70	2.23	**1.16'**	**0.19'**	**-0.61'**	**-1.28'**	-1.84
α and β unknown	9.83	8.20	6.55	4.96	3.51	2.23	**1.13'**	**0.20'**	**-0.58'**	**-1.23'**	-1.78

' and in bold: Non-rejection values at the 95% significance level.

TABLE 5										
Testing H_o (3) in (1) and (2) with AR(1) u_t										
z_t / d	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\alpha = \beta = 0$	---	--	--	--	1.77	**1.05'**	**0.49'**	**-0.37'**	**-1.16'**	-1.75	-2.19
α unknown $\beta = 0$	--	--	--	--	**1.13'**	**0.67'**	**0.26'**	**-0.52'**	**-1.27'**	-1.84	-2.26
$\alpha = 0$ β unknown	1.79	**0.21'**	**-0.06'**	**-0.35'**	**-0.65'**	**-0.97'**	**-1.28'**	-1.68	-1.87	-2.13	-2.37
α and β unknown	1.75	**-0.22'**	**-0.35'**	**-0.55'**	**-0.80'**	**-1.09'**	**-1.36'**	-1.67	-1.95	-2.20	-2.44

--- means that the test statistic does not decrease monotonically with respect to d_o. ' and in bold: Non-rejection values at the 95% significance level.

TABLE 6										
Testing H_o (3) in (1) and (2) with AR(2) u_t										
z_t / d	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\alpha = \beta = 0$	--	--	--	--	3.62	3.31	2.49	**1.32'**	**0.26'**	**-0.54'**	**-1.10'**
α unknown $\beta = 0$	--	--	--	--	2.85	2.48	2.06	**1.06'**	**0.06'**	**-0.70'**	**-1.24'**
$\alpha = 0$ β unknown	1.70	**1.34'**	**0.03'**	**-0.06'**	**-0.13'**	**-0.36'**	**-0.60'**	**-0.84'**	1.67	1.85	1.98
α and β unknown	1.98	**1.07'**	**-0.10'**	**-0.40'**	**-0.46'**	**-0.60'**	**-0.80'**	**-1.01'**	1.69	1.87	2.01

--- means that the test statistic does not decrease monotonically with respect to d_o. ' and in bold: Non-rejection values at the 95% significance level

TABLE 7											
Testing H_o (8) in (1) and (2)											
U_t / d	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
White noise	135726.01	67175.36	33261.33	16910.80	9189.69	5666.74	4247.62	3981.52	4424.82	5217.56	5800.63
AR(1)	135273.78	6674.27	32879.24	16627.12	9026.34	5596.75	4225.57	3977.52	4426.34	5220.32	5803.22
AR(2)	135335.14	66806.90	32955.46	16680.37	9027.05	5602.48	4229.77	3978.36	4424.7	5217.40	5799.65

The critical value corresponding to the χ_2^2 is 5.99 at the 95% significance level.