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Antonio Moreno

Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra
Antonio Moreno
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ABSTRACT

This paper has a twofold purpose. In the context of a structural macroeconomic model, it derives estimates of the Federal Reserve's preference parameters in its pre and post-1980 loss function. We show that there was an economically, but not statistically, significant change in the preferences of the U.S. Fed towards inflation stabilization. We also derive, within a strict inflation targeting regime, the optimal changes in the Fed's reaction to expected inflation as a function of the forward looking parameters in the supply and demand equations.

Antonio Moreno
Universidad de Navarra
Departamento de Economía
Campus Universitario
31080 Pamplona
antmoreno@unav.es
1 Introduction

There are now a number of papers which derive optimal monetary policy rules in the context of structural macroeconomic models.\footnote{Clarida, Galí, and Gertler (1999), Woodford (2002) or Lansing and Trehan (2003) are some examples.} One common feature of these works is that the resulting reduced form coefficients in the policy rule are composed of two sets of structural parameters, some pertaining to the monetary authority’s loss function and others describing the behavior of the private sector. While these papers derive normative prescriptions about the monetary authority behavior, very few estimates of the deep parameters in the monetary authority’s loss function are available in the literature. The present paper provides such estimates.

It has also been recently argued that there was a shift in the way monetary policy was conducted with the arrival of Paul Volcker to the Federal Reserve (Clarida, Galí, and Gertler (1999) and Boivin and Giannoni (2003) among others). The strategy followed by these researchers is based on estimating policy rules across sample periods and comparing the values of the long run coefficients on expected inflation. However, these empirical papers typically do not identify the Fed’s preferences embedded in the optimal policy rule coefficients.\footnote{Söderlind (1999) and Cecchetti and Ehrman (2000) obtain estimates of deep parameters. However, their work is not focused on the shift of the preferences of the U.S. Fed in the early 80s. Their methodology is also quite different to ours.} In this paper we show that there was indeed an economically significant change in the Fed’s preferences in the early 80s towards more inflation stabilization.

The second goal of this paper is to derive the optimal changes in the policy reaction function as the private sector becomes more forward-looking. This is a relevant question as papers by Boivin and Giannoni (2003) and Moreno (2003) have reported a significant increase of the private sector forward-looking behavior in the supply equation. To this end, we consider a standard three equation Rational Expectations macro model which exhibits endogenous persistence in the inflation rate, the output gap and the Federal funds rate. In the context of a strict inflation targeting framework, we show that the monetary authority should react more strongly to inflation as the price setting becomes more forward-looking up to a point. The reason is that when agents are very forward-looking, it is no longer effective to react more aggressively to inflation deviations from target when the price setting becomes more forward-looking. A similar finding is obtained in the case of the forward-looking parameter in the demand equation.
The paper proceeds as follows. First, we lay out the complete structural model which we consider. In the following section we define the monetary policy plan and obtain estimates of the Fed’s preferences in its loss function for different sample periods. In section 4 we derive the optimal changes in the Fed’s long run response coefficient on inflation as a function of the supply and demand forward-looking parameters. Section 5 concludes.

2 A Model for the US Economy

We first describe a simple macro model for the US economy which has been used in recent monetary policy studies such as Rotemberg and Woodford (1998). The model comprises supply, demand and monetary policy equations:

\[ \pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda y_t + \epsilon_{AS_t} \quad (1) \]
\[ y_t = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (i_t - E_t \pi_{t+1} - \bar{rr}) + \epsilon_{IS_t} \quad (2) \]
\[ i_t = \bar{rr} + \bar{\pi} + \rho i_{t-1} + (1 - \rho) [\beta (E_t \pi_{t+1} - \bar{\pi}) + \gamma y_t] + \epsilon_{MP_t} \quad (3) \]

\( \pi_t \) and \( y_t \) are the inflation rate and detrended output between time \( t-1 \) and \( t \) respectively, and \( i_t \) is the Federal funds rate at time \( t \). \( \bar{rr} \) is the long run natural real interest rate and \( \bar{\pi} \) is the long run level of inflation. One advantage of this macro model is that while being both parsimonious and structural, its implied dynamics are broadly consistent with those documented by empirical VAR studies.

The Aggregate Supply (AS) equation in (1) is a generalization of the Calvo (1983) pricing equation. The IS or demand equation in (2) can be derived through representative agent lifetime utility maximization as in Fuhrer (2000). Its endogenous persistence is due to the presence of habit formation in the utility function. Finally, the monetary policy equation in (3) is the one proposed by Clarida, Galí, and Gertler (2000). In this policy rule, the monetary authority smooths the interest rate path and reacts to expected inflation and to the output gap.

Table 1 shows the full information maximum likelihood (FIML) estimates obtained by the procedure described in Moreno (2003). We use Consumer Price Index (CPI) inflation, output detrended quadratically and the Federal funds rate. Given the evidence
of parameter instability documented in the literature, two sample periods were identified on the basis of the sup-Wald statistic derived by Bai, Lumsdaine, and Stock (1998). The sup-Wald test detects the fourth quarter of 1980 as the most likely break date in the parameters of an unconstrained vector autoregression. Accordingly, we start the second subsample on the fourth quarter of 1980.

Three major facts emerge from the parameter estimates. First, the three standard deviations of the structural shocks were lower in the second period, especially the one corresponding to the IS shock. This implies that macroeconomic conditions were more benign in the 80’s and 90’s. Second, the Fed reacted more strongly to expected inflation in the second period, although not significantly so. Third, private agents put more weight on expected inflation in the AS equation during the second period.

In the next section we will use these parameter estimates to obtain the probability distribution of the monetary authority’s preferences in its loss function across sample periods.

3 Estimates of the Preferences in the Loss Function of the Federal Reserve

Clarida, Galí, and Gertler (1999) first documented an increase in the Fed’s long run response to expected inflation after 1979. It is commonplace in the literature to see changes in this response arising from a shift in the preferences of the monetary authority. However, optimal monetary policy papers such as Woodford (2002) show that the long run response coefficients are also a function of structural private sector parameters. In this context, it could be that the Fed reacted more strongly to inflation after 1980 because it detected a modification in some structural parameters of the economy. In this section we show that the Fed’s more aggressive response to expected inflation was indeed the result of a change in its preferences.

We first formulate the optimization problem of the Federal Reserve which gives rise

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3 Both estimations yield a stationary Rational Expectations solution. The first period estimates imply multiple equilibria. In this instance we choose the equilibrium associated with the Recursive Method in Cho and Moreno (2003) which selects the bubble-free equilibrium.
to our interest rate reaction function:

$$\min_{i_t} \left\{ \frac{1}{2} \left[ \psi \lambda_\pi (E_t \pi_{t+1} - \bar{\pi})^2 + y_t^2 + \lambda_i i_t^2 + \lambda_\Delta (i_t - i_{t-1})^2 \right] \right\}$$  \hspace{1cm} (4)$$

subject to equations (1) and (2). \(\psi\) is the subjective time discount factor. \(\lambda_\pi\), \(\lambda_i\) and \(\lambda_\Delta\) are the objective function weights on expected inflation, interest rate variation and interest rate changes, respectively. This objective function implies that the Fed manages the nominal interest rate so as to stabilize deviations of expected inflation from its target as well as the current output gap. Additionally, the Fed tries to avoid excessive interest rate variation as well as deviations of the current interest rate from its past value. Since this objective function does not contain expected future terms beyond the next period inflation forecast, this optimization problem falls under the category of discretion. In other words, the Fed reoptimizes every period the objective function in (4) in order to choose the value of its instrument, the Federal funds rate.\(^4\)

The implied interest rate rule becomes, in mean deviation form:

$$i_t = \frac{\lambda_\Delta}{\lambda_i + \lambda_\Delta} i_{t-1} - \psi \lambda_\pi \frac{\partial E_t \pi_{t+1}}{\partial i_t} E_t \pi_{t+1} - \frac{1}{\lambda_i + \lambda_\Delta} \frac{\partial y_t}{\partial i_t} y_t$$  \hspace{1cm} (5)$$

Matching these coefficients with those in the Clarida, Galí, and Gertler (2000) rule in (3), which we estimated, yields:

$$\beta = - \frac{\psi \lambda_\pi \partial E_t \pi_{t+1}}{\lambda_i} \frac{\partial \pi_{t+1}}{\partial i_t}$$  \hspace{1cm} (6)$$

$$\gamma = - \frac{1}{\lambda_i} \frac{\partial y_t}{\partial i_t}$$  \hspace{1cm} (7)$$

$$\rho = \frac{\lambda_\Delta}{\lambda_i + \lambda_\Delta}$$  \hspace{1cm} (8)$$

Equation (6) shows that as the Fed puts more weight on expected inflation deviations from target, it responds more aggressively to expected inflation in the policy rule. The partial derivatives in the policy rule coefficients reflect the constraints imposed by the

\(^4\)Svensson (2003) criticizes forecast-based instrument rules of this kind on the grounds of time inconsistency. Our goal in this paper is to provide a deeper interpretation of the coefficients in an interest rate rule which seems to capture the short term interest rate dynamics quite closely. Accordingly, the loss function which we consider does not include any term of periods other than the current one. This precludes the appearance of the term \(E_t i_{t+1}\) as an argument in the reaction function.
AS and IS equations in the optimization program of the monetary authority. Equations (6) and (7) show that it is optimal to react more strongly to expected inflation and the output gap when these two variables become more sensitive to changes in the interest rate, respectively. Notice that the partial derivatives \( \frac{\partial E_t \pi_{t+1}}{\partial i_t} \) and \( \frac{\partial y_t}{\partial i_t} \) will be negative for reasonable parameter values as obtained in our estimations. Equation (8) shows that the optimal coefficient on the interest rate lag, \( \rho \), grows in tandem with the tendency of the Fed towards smoothing the interest rate. Finally, when the Fed puts less weight on interest rate variability the three policy coefficients become larger.

One difficulty in computing the partial derivatives in our setting is the endogeneity of the interest rate, \( i_t \), in our model. However, given our policy rule, we can distinguish an exogenous part as a source of interest rate fluctuations, the monetary policy shock \( \epsilon_{MP} \). In Appendix A we derive these partial derivatives by proxying changes in \( i_t \) with changes in its exogenous part. Hence, we can identify \( \lambda_\pi \), \( \lambda_i \) and \( \lambda_\Delta \) uniquely by setting \( \psi \) to 0.99, a standard value in the literature. In order to compute the differences in the Fed’s loss function weights across sample periods, we use the parameter estimates shown in Table 1.

The estimates in Table 2 show that there was an increase in \( \lambda_\pi \), implying a larger concern of the Fed about deviations of expected inflation from its target. Accordingly, the larger estimate of the long run response to expected inflation can indeed be economically attributed to changes in the preferences of the monetary authority, and not to changes in the structural parameters in the AS and IS equations.\(^5\) Table 2 also lists the implied monetary policy weights on the expected inflation, interest rate and interest rate difference terms for both periods. The point estimates show that the Fed was less concerned about interest rate variability in the second period, since the term \( \lambda_i \) became smaller in the 80’s and 90’s. \( \lambda_\Delta \) is larger in the second period, reflecting an increasing concern of the monetary authority towards smoothing the interest rate path.

In order to draw statistical inference, we compute the probability distributions of the policy parameters. To this end, we perform a Montecarlo simulation by taking draws from the distributions of the structural parameters and computing the policy preferences. We repeat this exercise 1000 times, yielding a probability distribution for

\(^5\)In a related exercise within a more stylized framework, Cecchetti and Ehrman (2000) show that the Central Banks of several countries which adopted inflation targeting put more weight on inflation deviations after the adoption date.
the policy weights. Explosive solutions to the structural model and negative values for the policy preferences were discarded in the process. The first set of 90% confidence intervals appears in parentheses below the point estimates in Table[2] It shows that there is plenty of statistical uncertainty regarding the values of the preference parameters, especially in the second period. In our exercise, this uncertainty stems from the combined uncertainty of the structural parameters, which is especially pronounced in the monetary transmission mechanism parameters, $\lambda$ and $\phi$. We then compute confidence intervals fixing these two parameters. They appear in square brackets. While the intervals are still quite large, they are clearly tighter than in the first case. For instance, the weight on expected inflation is significantly larger in the second period. This finding implies that much of the uncertainty in the policy preferences is due to the large standard errors of $\lambda$ and $\phi$.

To summarize the results in this section, we showed that there was an important shift in the preferences of the Fed since 1980 in favor of inflation stabilization. This date approximately coincides with the arrival of Paul Volcker to the Federal Reserve Board. We also showed that this result is not statistically significant. This uncertainty was shown to stem mainly from the imprecise estimates of the parameters describing the monetary policy transmission mechanism.

4 Optimal Policy Under Endogenous Persistence

Our macroeconomic model exhibits endogenous persistence in the supply and demand equations. This endogenous persistence is captured by the private sector parameters $(1-\delta)$ and $(1-\mu)$ in the structural model. As $\delta$ and $\mu$ become larger, inflation and output display less persistency. As shown in the previous section, the optimal response of the Fed to expected inflation depends on all the parameters of the structural model. In this section we assess the impact of changes in these private sector parameters on the optimal Fed's response to expected inflation. This is a relevant issue, since the estimates in Boivin and Giannoni (2003) and Moreno (2003) show a significant increase in $\delta$, the forward-looking parameter in the AS equation, during the 80s and 90s.

In order to determine unique optimal relations between $\beta$ and $\delta$ and between $\beta$ and $\mu$, we will assume in our analysis that the monetary authority practices strict inflation targeting, i.e. $\gamma = 0$. In Appendix B, we show how to derive the optimal relation between
the long run response to inflation and the forward-looking parameters through numerical approximations. These approximations can be computed via the implicit function theorem, since the partial derivatives in (6) depend on $\beta$ and both $\delta$ and $\mu$ . As shown in this appendix, the sign relation between the Fed’s response to inflation and these forward-looking parameters can be expressed as:

$$\frac{\partial \tilde{\beta}}{\partial \theta_i} = -\frac{k \cdot \frac{\partial f(\theta)}{\partial \theta_i}}{1 + k \cdot \frac{\partial f(\theta)}{\partial \beta}}$$

where $\tilde{\beta}$ is an implicit function of $\beta$.

The top panel of Figure 1 graphs $f(\theta)$ as a function of $\beta$ for different values of $\lambda$, including the estimated one, 0.0072. The remaining AS and IS parameters are fixed at their first period estimates, whereas the policy parameters are fixed at their second period values. Results do not change if we fix all the remaining parameters at either their 1st or 2nd period values. The top panel of Figure 1 shows that, for realistic parameter values ($\beta > 0.2$), $f(\theta)$ is an increasing function of $\beta$. This implies that the denominator in (9) will always be positive for a reasonable parameter space. Hence the sign of the implicit function’s partial derivative, $\frac{\partial \tilde{\beta}}{\partial \theta_i}$, depends inversely on the sign of the partial derivative $\frac{\partial f(\theta)}{\partial \theta_i}$.

Panel B of Figure 1 shows that $f(\theta)$ is initially a decreasing function of $\delta$. However, when $\delta$ becomes larger (approximately 0.6), $f(\theta)$ becomes and increasing function of $\delta$. This implies that for values of $\delta$ smaller than 0.6 the Fed should react more strongly to expected inflation as agents become more forward-looking in the supply equation. However, when $\delta$ is sufficiently large, the Fed should respond less aggressively.

To understand the intuition behind this result, recall that the loss function in (5) implies that the Fed will react more strongly to expected inflation when changes in the interest rate are more effective in reducing expected inflation. Our structural macro model contains two monetary transmission mechanisms from the interest rate to inflation. Interest rate changes affect the output gap through the real rate in the IS equation. In turn, output gap movements influence inflation through the Phillips curve relation in the AS equation. The second channel consists of expectational effects. As the monetary authority reacts more aggressively to inflation, the private sector adjusts its inflation expectations which affect directly the inflation rate. We now show how the effectiveness
of monetary policy varies with the endogenous persistence of inflation and the output gap.

At small values of $\delta$, contractionary monetary policy is quite effective in reducing expected inflation, as future inflation still depends heavily on current inflation. Hence as agents become more forward-looking when $\delta$ is still not very large, current inflation will experience a larger reduction following an interest rate increase (since the term $\delta E_t \pi_{t+1}$ in the supply equation will be larger in absolute value). This reinforces the decline in expected inflation making contractionary monetary policy more effective. However, at high values of $\delta$, expected inflation does not greatly depend on current inflation. Accordingly, increases in $\delta$ will make expected inflation even less dependent on future inflation (and will also make the term $\delta E_t \pi_{t+1}$ smaller in absolute value) so that monetary policy will be less effective in reducing expected inflation. This implies the existence of a cutoff value $\delta^*$ such that for $\delta > \delta^*$, it is no longer optimal to react more aggressively to expected inflation as $\delta$ grows.

Panel B of Figure 1 performs a calibration exercise around different values of the Phillips curve parameter $\lambda$. It shows that when monetary policy is less effective in reducing inflation volatility (for smaller values of $\lambda$), the cutoff value is smaller, whereas for larger values of $\lambda$, the cutoff value is higher. Finally, the top panel of Figure 2 presents the impulse responses of inflation to a monetary policy shock. It shows that for low values of $\delta$ in the AS equation, as agents become more forward-looking, monetary policy is more effective in reducing inflation for about 15 periods. However, when $\delta$ is very large, the opposite is true during the 50 periods following the shock. These two pieces of evidence corroborate the ideas in the previous paragraph.

Panel C of Figure II graphs $f(\theta)$ as a function of $\mu$, the forward-looking parameter in the IS equation. It is decreasing up to values of $\mu$ close to 0.4, when it starts to increase. As a result, for values of $\mu$ close to 0.5, as estimated in our sample, increases in the forward-looking behavior of agents in the IS equation should be followed by a smaller reaction of the Fed to expected inflation. The reason is that for large values of $\mu$, a smaller output gap persistence will make monetary policy less effective, since the way it influences inflation is by contracting the output gap. However, when $\mu$ is below 0.4, increases in the forward-looking parameter in the IS equation will reinforce the effect of contractionary monetary policy on the current output gap (through a larger value of

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6We stress that our results are limited to the case of strict inflation targeting.
the $\mu E_t y_{t+1}$ term in the IS equation). The mechanism is analogous to that of $\delta$ and is illustrated on Panel B of Figure 2. When $\mu$ is small, as it grows, monetary policy becomes more effective in reducing inflation, whereas the opposite is true when $\mu$ is large.

Our results resemble those obtained by Lansing and Trehan (2003) in a related optimal monetary policy exercise. There are however some differences. In their case, as $\mu$ grows, it is optimal to react less aggressively to inflation as long as $\mu$ is greater than 0.1, whereas in our paper the cutoff value $\mu^*$ is 0.4. While Lansing and Trehan (2003) include output gap stabilization in their central bank loss function, they also consider a different structural model. Whereas in the present paper the expectations influence all the variables contemporaneously, in their case it is the lagged expectations which affect the current period variables. This implies that in our exercise the expectations and the current values affect each other simultaneously, so that as $\mu$ grows, monetary policy is more effective up to higher values of $\mu$ (since the current output gap will react more strongly to interest rate changes). Our results also differ in the case of optimal monetary policy for different values of $\delta$. In Lansing and Trehan (2003) it is not until $\delta$ is around 0.95 that increases in $\delta$ should restrain the monetary policy reaction, whereas in our case the cutoff value is around 0.60. This occurs because monetary policy starts being ineffective earlier in our case.

To understand the different optimal Fed’s behavior under increases of $\mu$ and $\delta$ in our exercise ($\mu^*$ is larger with respect to Lansing and Trehan (2003) whereas $\delta^*$ is smaller), notice that a given increase in a parameter results in a larger percentage increase at small parameter values. This effect dominates at small values of $\mu$, where initial increases of $\mu$ have a sizable effect on the output gap, with the output gap being still quite persistent. However, in the case of $\delta$, when this parameter is quite large, further increases of $\delta$ would have two effects. On the one hand, the percentage increase in $\delta$ is small and on the other hand, it exacerbates the already small degree of persistence in inflation, making expected inflation depend even less on current inflation. These two effects are amplified in our exercise, where expectations affect the macro variables contemporaneously.

The results in this section illustrate the importance of the expectational effects in monetary policy management. As the private sector becomes more forward-looking, expectations of the future variables behave differently, so that the effects of monetary policy actions also differ. One important implication of our study is that the monetary authority should not modify its reaction to inflation monotonically as changes in the
private sector behavior occur.

5 Conclusion

This paper shows that there was an economically, but not statistically, significant change in the preferences of the Federal Reserve after 1980 towards inflation stabilization. We also show, in the context of a strict inflation targeting regime, the optimal changes in the Fed’s reaction to expected inflation when the forward-looking behavior of the private sector changes.

The importance of the private sector’s degree of forward-looking behavior has been highlighted in this paper. There have been several attempts in the literature to derive aggregate supply equations featuring both forward and backward looking components. Fuhrer and Moore (1995), for instance, develops a real wage contracting model with endogenous persistence. In this case, the persistence is induced by the existence of wage-setters who adjust their current real wages with respect to past real wages. One caveat of these works is that the endogenous persistence of inflation is not ultimately grounded in optimizing behavior. A better understanding of the sources of inflation persistence would be desirable as the Fed’s optimal policy changes with it.
Appendix

A Computing the Partial Derivatives

Equations (6) and (7) show that the optimal $\beta$ and $\gamma$ coefficients in our reaction function depend on partial derivatives of the target variables with respect to interest rate changes. In order to compute these partial derivatives, we recognize both an endogenous and an exogenous part in our model’s interest rate:

$$i_t = \tilde{i}_t + \check{i}_t$$

(10)

where, in mean deviation, $\tilde{i}_t = \rho i_{t-1} + (1 - \rho)(\beta E_t \pi_{t+1} + \gamma y_t)$ and $\check{i}_t = \epsilon_{MP_t}$. Therefore, $\tilde{i}_t$ constitutes the exogenous part. In this setting, we can proxy the partial derivative terms involving changes in $i_t$ with changes in $\tilde{i}_t$ by applying vector differentiation rules to our model solution. The implied model’s solution is: $\bar{X}_{t+1} = \Omega \bar{X}_t + \Gamma \epsilon_{t+1}$, where $\bar{X}_t = [\pi_t \ x_t \ i_t]'$ in demeaned form. Since the next period expectations can be expressed as $E_t \bar{X}_{t+1} = \Omega \bar{X}_t$, we can obtain:

$$E_t \bar{X}_{t+1} = \Omega(\Omega \bar{X}_{t-1} + \Gamma \epsilon_t)$$

(11)

Therefore $\frac{\partial E_t \bar{X}_{t+1}}{\partial \epsilon_t} = \Omega \Gamma$, so that $\frac{\partial E_t \pi_{t+1}}{\partial i_t}$ is simply the (1,3) element of the product matrix $\Omega \Gamma$, i.e.:

$$\frac{\partial E_t \pi_{t+1}}{\partial i_t} = \Omega_{11} \Gamma_{13} + \Omega_{12} \Gamma_{23} + \Omega_{13} \Gamma_{33}$$

(12)

In order to obtain the partial derivative $\frac{\partial y_t}{\partial i_t}$ in equation (7), we follow the procedure described above and use the IS equation to obtain:

$$\frac{\partial y_t}{\partial i_t} = \Gamma_{13}(\mu \Omega_{21} + \phi \Omega_{11}) + \Gamma_{23}(\mu \Omega_{22} + \phi \Omega_{12}) + \Gamma_{33}(\mu \Omega_{23} + \phi(1 - \Omega_{13}))$$

(13)

B Optimal Fed Behavior

In this second part of the Appendix, we derive the optimal changes in the Fed’s reaction to expected inflation when the behavior of the private sector in the AS and IS equations
varies. We reproduce equation (6) for ease of exposition:

$$\beta = -\frac{\psi \lambda_\pi}{\lambda_i} \pi_t \partial E_t \pi_{t+1} \partial i_t$$

(14)

In the previous appendix, we computed an approximation to the term \(\frac{\partial E_t \pi_{t+1}}{\partial i_t}\). As can be seen in equation (12), this term depends on the reduced form elements of the model’s Rational Expectations solution which, in turn, also depend on \(\beta\). Therefore, we can apply the Implicit Function theorem to equation (14) so as to determine how the Fed would change its reaction to expected inflation when the private sector becomes more forward looking in the supply and demand equations.

Let us first introduce some additional notation:

$$f(\theta) = \frac{\partial E_t \pi_{t+1}}{\partial i_t} \delta, \lambda, \mu, \phi, \beta, \gamma, \rho \in \theta$$

(15)

Then, we can rewrite (14) as:

$$\tilde{\beta} = \beta + k \cdot f(\theta) = 0 \quad k = \frac{\psi \lambda_\pi}{\lambda_i} > 0$$

(16)

Using the Implicit Function theorem:

$$\frac{\partial \tilde{\beta}}{\partial \theta_i} = -\frac{k \cdot \frac{\partial f(\theta)}{\partial \theta_i}}{1 + k \cdot \frac{\partial f(\theta)}{\partial \beta}} \forall \theta_i \in \theta$$

(17)

In order to obtain the sign of the partial derivatives, we can compute numerically vectors of the \(\Omega_{ij}\) and \(\Gamma_{ij}\) terms in (12) as a function of \(\delta, \mu\) and \(\beta\) holding the remaining parameters constant. In this way, we can graphically identify the sign of the terms \(\frac{\partial f}{\partial \theta_i}, \frac{\partial f}{\partial \beta}\) and, in turn, the sign of the derivative \(\frac{\partial \beta}{\partial \theta_i}\) for \(\delta, \mu \in \theta_i\). Figure 11 graphs the functions involved in the implicit function derivatives in (17).
References


Table 1: FIML estimates

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<th></th>
<th>1st Period</th>
<th>2nd Period</th>
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<td>(\delta)</td>
<td>0.5482</td>
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<td></td>
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<td>(\lambda)</td>
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<td></td>
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<td></td>
<td>(0.0213)</td>
<td>(0.0219)</td>
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<tr>
<td>(\phi)</td>
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<td></td>
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<td></td>
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<td>(\gamma)</td>
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<td>(\sigma_{AS})</td>
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<td></td>
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<tr>
<td>(\sigma_{IS})</td>
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<td></td>
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<td>(0.0564)</td>
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This Table shows the FIML parameter estimates of the structural New Keynesian macro model obtained by the procedure outlined in Moreno (2003). CPI inflation, output detrended quadratically and the Federal funds rate are the variables used in estimation. The first subsample spans the period 1957:2Q-1980:3Q and the second subsample covers the period 1980:4Q-2001:2Q. The model’s equations in demeaned form are:

\[
\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda y_t + \epsilon_{AS_t},
\]

\[
y_t = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (r_t - E_t \pi_{t+1}) + \epsilon_{IS_t},
\]

\[
r_t = \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{MP_t}.
\]
Table 2: **Implied weights of the U.S. Fed in its Objective Function**

<table>
<thead>
<tr>
<th>Panel A: CPI</th>
<th>1st Period</th>
<th>2nd Period</th>
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<tbody>
<tr>
<td>(\lambda_\pi)</td>
<td>1.2937</td>
<td>6.1078</td>
</tr>
<tr>
<td>(0.7415 28.4476)</td>
<td>(1.0966 122.2351)</td>
<td></td>
</tr>
<tr>
<td>[0.9066 3.5010]</td>
<td>[3.1023 72.8869]</td>
<td></td>
</tr>
<tr>
<td>(\lambda_i)</td>
<td>0.1039</td>
<td>0.0599</td>
</tr>
<tr>
<td>(0.0363 0.4079)</td>
<td>(0.0192 0.3810)</td>
<td></td>
</tr>
<tr>
<td>[0.0675 0.3131]</td>
<td>[0.0214 0.1246]</td>
<td></td>
</tr>
<tr>
<td>(\lambda_\Delta)</td>
<td>0.3545</td>
<td>0.4334</td>
</tr>
<tr>
<td>(0.1112 2.1532)</td>
<td>(0.1211 3.5913)</td>
<td></td>
</tr>
<tr>
<td>[0.1796 1.5693]</td>
<td>[0.2980 2.2491]</td>
<td></td>
</tr>
</tbody>
</table>

This Table lists the preference parameters in the objective function of the Federal Reserve in (4) across sample periods. \(\lambda_\pi\) is the weight on expected inflation, \(\lambda_i\) is the weight on the interest rate and \(\lambda_\Delta\) is the weight on the interest rate first difference. Two sets of 90% confidence intervals obtained through a Montecarlo simulation appear in parenthesis and square brackets below the point estimates. The first set excludes both explosive model’s solutions and negative values for the Fed’s preferences in the Montecarlo study. The second set additionally fixes the parameters \(\lambda\) and \(\phi\) in the simulations.
Panel A: Denominator slope ($f(\beta)$)

Panel B: Numerator slope ($f(\delta)$)

Figure 1: $f(\theta)$ functions used to compute the Implicit Function Derivative
Panel C: Numerator slope \((f(\mu))\)

Figure 1: (continued) \(f(\theta)\) functions used to compute the Implicit Function Derivative

Panel A graphs the function in (15) \((f(\theta))\) depending on \(\beta\) and holding the remaining parameters fixed. Panel B graphs it as a function of \(\delta\) and Panel C graphs it as a function of \(\mu\). The slope of these functions determine the signs of the numerator and denominator of the Implicit Function derivative in equation (17).
Panel A: AS Equation Forward Looking Parameter

Panel B: IS Equation Forward Looking Parameter

Figure 2: Inflation Responses to a Monetary Policy Shock

This figure presents the response functions of inflation to a monetary policy shock. Panel A presents responses under alternative values of $\delta$, whereas Panel B shows inflation responses for different values of $\mu$. The remaining model’s parameters are held at their second period estimates.