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Reaching Inflation Stability

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ABSTRACT

Inflation volatility has significantly declined over the last 20 years in the U.S. To find out why, I follow a structural approach. I estimate a complete New Keynesian model which imposes cross-equation restrictions on the time series of inflation, the output gap and the interest rate. I perform counterfactual analysis with the most commonly used measures of inflation: Consumer Price Index (CPI) and Gross Domestic Product Deflator (GDPD). While the change in the propagation mechanism of the economy induced most of the CPI volatility drop, it played a smaller role in the reduction of GDPD volatility. Our maximum likelihood estimates imply that the most important factor behind the drop in inflation volatility was the more forward-looking price setting behavior of the 80s and 90s.

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1 Introduction

Inflation and its variability entail large real costs to the economy. Several studies show that a 10% inflation rate can produce losses of around 3% of the real GNP through saving and investment misallocation or the loss of value of real balances.\(^1\) In light of these figures, the current era of low inflation level and volatility constitutes a major macroeconomic development. In this paper we attempt to identify the driving forces which led to the current low inflation volatility in the United States.\(^2\)

The top panel of Figure 1 graphs historical inflation series since 1957. They were calculated with three measures of the price level frequently used by researchers and policy makers: the Consumer Price Index (CPI), the Personal Consumption Expenditures Deflator (PCE) and the GDP Deflator (GDPD). The figure shows the steady increase of all the inflation series since the mid-60s up to the beginning of the 80s. Since then, inflation has been drastically reduced. Current inflation hovers at low levels, comparable to those of the early 60s. One difference among the series is that during our sample period, GDPD inflation peaks at the end of 1974, after the first oil shock, whereas the CPI and PCE inflation rates reach their maximum values in the beginning of the 80s. The bottom panel of Figure 1 graphs a 20 quarter rolling standard deviation of each inflation measure. It also shows an important increase since the mid 60s followed by a steady decline starting in the early 80s. Table 1 lists the sample statistics of the inflation series for different sample periods. All the series have experienced a large drop in averages and volatilities since the end of 1980. While the three measures of inflation exhibit smaller first order autocorrelations during the second period, this decline is less pronounced in the case of the GDPD. Table 1 shows that these empirical facts are reinforced if we remove the observations included in the high inflation volatility period which goes from 1978 to 1983. The overall picture of lower volatility which emerges from Table 1 and Figure 1 motivates the central question in this paper: What led to a lower inflation variability?

The approach followed in this paper to answer this question is based upon two building blocks. First, we formulate a monetary New Keynesian model of the macroeconomy which comprises aggregate supply, aggregate demand and monetary policy rule equa-


\(^2\)While the present paper focuses on the drop of inflation volatility, our results also apply to the reduction in the level of inflation. A robust positive relation between the level and the variance of inflation has been long documented in the literature: Okun (1971), Friedman (1977) or Taylor (1981).
tions with endogenous persistence. In this model higher rates of inflation trigger the response of the monetary authority which raises interest rates. Changes in the real rate, in turn, reduce the welfare of the representative agent. The introduction of a model has the advantage that it allows us to identify specific propagation mechanisms of structural shocks. The New-Keynesian model seems adequate for our exercise, as it implies macroeconomic dynamics which represent a good approximation to those observed in the data, as shown by Rotemberg and Woodford (1998) and others. In fact, our model estimates yield standard deviations and autocorrelation patterns which are broadly consistent with those found in the data.

Second, we develop a counterfactual analysis in order to determine the driving forces behind the current low inflation environment. This methodology is particularly useful for our task, as it makes the private agents and the monetary authority confront shocks of different sample periods. Hence, it reveals the counterfactual inflation volatilities which would have arisen under different combinations of macroeconomic conditions (shocks) and private sector/monetary policy behavior. In this way, we can determine what factors were instrumental in the reduction of inflation volatility.

Our counterfactual analysis also assumes that there is a sudden shift in the structural model parameters and that both the private sector and the monetary authority recognize it immediately. This is a limitation of our framework, since shifts which agents perceived with probability zero just before the break are perfectly understood right after. However, we think that our approach can be seen a first order approximation to what happened in reality, where agents assign probabilities to parameter changes. Additionally, in order to check for the robustness of our results, we perform a sensitivity analysis around the estimated parameter values.

We impose the model’s implied cross-equation constraints in estimation and perform alternative estimations with the three inflation measures: CPI and GDPD inflation. We find that while CPI inflation volatility fell because the internal propagation mechanism changed, the lower shocks had a large impact on the decline of GDPD inflation. We show some evidence pointing to the prices of investment goods, specially those of equipment, as responsible for this differences.

Our maximum likelihood estimates imply that the change towards the more forward-looking price setting of the 80s and 90s was the most influential factor in the change of
the propagation mechanism. We find that the shift towards a more aggressive monetary policy rule of the last two decades also mattered, but to a lesser extent. One implication of our study is the need to understand better the sources of changes in the price setting. It could be that the “forward-lookingness” of the price-setting process is related to monetary policy, but the New-Keynesian model is, in principle, silent about it.

The literature on the drop of inflation volatility is quite recent. Boivin and Giannoni (2002) and Ahmed, Levin, and Wilson (2002) use a VAR approach to determine whether the decrease of inflation volatility over the last 20 years was due to smaller shocks or to changes in the overall transmission mechanism of these exogenous disturbances. The structural approach followed in this paper allows us to determine the origin of the changes in the transmission mechanism. Unlike a less structured approach, we can determine whether variations in the propagation mechanism were due to a change in the conduct of monetary policy or to parameters describing the structure of the economy. We also perform a more comprehensive analysis of the drop of inflation volatility, as we look at both CPI and GDP.

Our paper is also related to the literature on parameter stability of structural macro models. Clarida, Galí, and Gertler (1999) and Boivin and Giannoni (2003) detected a significant increase in the response of the Fed to inflation after Volcker’s arrival. Additionally, Bernanke and Mihov (1998) find parameter instability in their identified VARs, whereas Ireland (2001) finds parameter instability in a structural New-Keynesian macro model. In this paper we explore whether, and which, changes in structural parameters triggered the decline of inflation volatility.

Two closely related papers are Stock and Watson (2002) and Cogley and Sargent (2002). The first paper uses a structural approach similar to the one employed in this paper, but its goal is to uncover the factors behind the decline of output volatility. In contrast to their study, we let all of the structural parameters vary across periods and not just those in the policy rule. This difference turns out to be critical in our case, as we detect a significant change in the forward-looking parameter of the AS equation. Finally, Cogley and Sargent (2002) estimate a time-varying parameter model and find a clear inverse relation between the persistence of CPI inflation and the Fed’s degree of responsiveness to inflation. While our model estimates also capture this contemporaneous relation, we show that other factors, such as the forward-looking price setting of the 80s and 90s, may have also been influential.

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This paper proceeds as follows. Section 2 lays out the model of the economy. Section 3 discusses the Rational Expectations solution associated with the model. Section 4 describes the data and the estimation procedure. In Section 5 we perform the break date tests in order to identify two separate subsamples. In Section 6 we show our main results. Section 7 concludes.

2 A Macro Model for the U.S. Economy

This section lays out a simple linear Rational Expectations model of the macroeconomy which is similar to the ones employed in recent studies of monetary policy such as Rotemberg and Woodford (1998). The model comprises aggregate supply (AS), aggregate demand (IS) and monetary policy equations. The derivations of each of the equations are consigned to the Appendix.

The aggregate supply equation is a generalization of the supply specification originally developed by Calvo (1983):

\[ \pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda y_t + \varepsilon_{AS_t} \]  

\( \pi_t \) is inflation between \( t-1 \) and \( t \) and \( y_t \) stands for the output gap between \( t-1 \) and \( t \). \( \varepsilon_{AS_t} \) is the aggregate supply structural shock, assumed to be independently and identically distributed with homoskedastic variance \( \sigma_{AS}^2 \). It can be interpreted as a cost push shock which makes real wages deviate from their equilibrium value or simply as a pricing error. \( E_t \) is the Rational Expectations operator conditional on the information set at time \( t \), which comprises \( \pi_t, y_t, r_t \) (the nominal interest rate at time \( t \)) and all the lags of these variables. Equation (1) shows that \( \delta \) grows as the private sector puts more weight on expected inflation. A virtue of this pricing specification is that it captures the empirical properties of U.S. inflation dynamics quite accurately. As the Appendix makes clear, the endogenous persistence arises due to the existence of price setters who do not adjust optimally and index their prices with respect to past inflation.

The IS or demand equation is based on representative agent intertemporal utility
maximization with external habit persistence, as proposed by Fuhrer (2000):

\[ y_t = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (r_t - E_t \pi_{t+1}) + \varepsilon_{IS,t} \]  

(2)

where \( \varepsilon_{IS_t} \) is the IS shock, assumed to be independently and identically distributed with homoskedastic variance \( \sigma^2_{IS} \). In our specification, it is the habit formation specification in the utility function which imparts endogenous persistence to the output gap. The monetary policy channel in the IS equation is captured by the contemporaneous output gap dependence on the ex ante real rate of interest. This relation arises in standard Euler equations derived by lifetime utility maximization. The monetary transmission mechanism depends negatively on the curvature parameter in the utility function, \( \sigma \) and, for \( \sigma > 1 \), on the parameter that indexes habit persistence, \( h \), since \( \phi = \frac{1}{\sigma(1+h)-h} \). \( \sigma \) represents the inverse of the elasticity of substitution in the absence of habit formation. Appendix A.1 shows that \( \varepsilon_{IS_t} \) is proportional to the utility function disturbances.

We close the model with the monetary policy rule formulated by Clarida, Galí, and Gertler (2000):

\[ r_t = \alpha_{MP} + \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \varepsilon_{MP,t} \]  

(3)

\( \alpha_{MP} \) is a constant and \( \varepsilon_{MP,t} \) is the monetary policy shock, assumed to be independently and identically distributed with homoskedastic variance \( \sigma^2_{MP} \). The policy rule has two well differentiated parts. On the one hand, the monetary authority smooths interest rates, placing a weight of \( \rho \) on the past interest rate. On the other hand, it reacts to high expected inflation and to deviations of output from its trend. The parameter \( \beta \) measures the long run response of the Central Bank to expected inflation, whereas \( \gamma \) describes its reaction to output gap fluctuations. We assume that the Federal funds rate is the monetary policy instrument, as much of the previous literature does.
3 Rational Expectations Equilibrium

3.1 Model Equilibrium and Implications

In this section we follow the framework laid out in Cho and Moreno (2002) to derive the Rational Expectations equilibrium of the model. Our macroeconomic system of equations (1), (2) and (3) can be expressed in matrix form as follows:

\[
\begin{bmatrix}
1 & -\lambda & 0 \\
0 & 1 & \phi \\
0 & (1-\rho)\gamma & 1 \\
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_t \\
r_t \\
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & \alpha_{MP} \\
\delta & 0 & 0 \\
\phi & \mu & 0 \\
(1-\rho)\beta & 0 & 0 \\
\end{bmatrix}
E_t
\begin{bmatrix}
\pi_{t+1} \\
y_{t+1} \\
r_{t+1} \\
\end{bmatrix}
+ 
\begin{bmatrix}
1 & -\delta & 0 \\
0 & 1-\mu & 0 \\
0 & 0 & \rho \\
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
y_{t-1} \\
r_{t-1} \\
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{AS_t} \\
\varepsilon_{IS_t} \\
\varepsilon_{MP_t} \\
\end{bmatrix}
\]

In more compact notation:

\[
B_{11}X_t = \alpha + A_{11}E_tX_{t+1} + B_{12}X_{t-1} + \varepsilon_t, \varepsilon_t \sim (0, D) \tag{4}
\]

where \(X_t = (\pi_t, y_t, r_t)'\), \(B_{11}, A_{11}\) and \(B_{12}\) are the coefficient matrices of structural parameters, and \(\alpha\) is a vector of constants. \(\varepsilon_t\) is the vector of structural errors, \(D\) is the diagonal structural error variance matrix and 0 denotes a \(3 \times 1\) vector of zeros. Following a standard Undetermined Coefficients approach, the Rational Expectations equilibrium to the system in (4) can be written as the following reduced-form:

\[
X_{t+1} = c + \Omega X_t + \Gamma \varepsilon_{t+1} \tag{5}
\]

where \(c\) is a \(3 \times 1\) vector of constants and \(\Omega\) and \(\Gamma\) are \(3 \times 3\) matrices. To see this, substitute equation (5) into equation (4) and rearrange by applying Rational Expectations. Then:

\[
(B_{11} - A_{11}\Omega)X_t = \alpha + A_{11}c + B_{12}X_{t-1} + \varepsilon_t \tag{6}
\]

Since the three structural equations are linearly independent, \((B_{11} - A_{11}\Omega)\) is nonsingular. Then, pre-multiplying by \((B_{11} - A_{11}\Omega)^{-1}\) on both sides in equation (6) and matching the
coefficient matrices of $X_{t-1}$ and $\varepsilon_t$, we obtain:

\[
\Omega = (B_{11} - A_{11} \Omega)^{-1} B_{12} \\
\Gamma = (B_{11} - A_{11} \Omega)^{-1} \\
c = (B_{11} - A_{11} \Omega - A_{11})^{-1} \alpha
\] (7) (8) (9)

Therefore, equation (5) with $\Omega$, $\Gamma$ and $c$ satisfying equations (7), (8) and (9) is a solution to equation (4). Once we solve for $\Omega$ as a function of $A_{11}$, $B_{11}$ and $B_{12}$, $\Gamma$ and $c$ can be easily calculated. Notice that the implied reduced-form of our structural model is simply a VAR of order 1 with highly nonlinear parameter restrictions. There is a linear relation between the structural errors, $\varepsilon_t$ and the reduced-form Rational Expectations errors ($v_t$), through $\Gamma$,

\[
v_t = \Gamma \varepsilon_t
\] (10)

The Rational Expectations equilibrium also yields a simple linear relation between $\Omega$ and $\Gamma$ through $B_{12}$, which captures the dependence of the system on the lagged predetermined variables:

\[
\Omega = \Gamma B_{12}
\] (11)

### 3.2 Characterization of the Rational Expectations Equilibrium

We will utilize two methods in order to determine the Rational Expectations equilibrium to our system. First, we will use the generalized Schur matrix decomposition method (QZ) developed by Klein (2000) and outlined by McCallum (1999) in order to obtain the Rational Expectations equilibrium. The QZ method yields a solution even when the matrix $A_{11}$ is singular, which is the case in our model. Appendix B.1 describes the derivation of the Rational Expectations Solution through the QZ method.

For $\Omega$ satisfying (4) to be admissible as a solution, it must be real-valued and exhibit stationary dynamics. Because $\Omega$ is a nonlinear function of the structural parameters in $B_{11}$, $A_{11}$ and $B_{12}$, there could potentially be multiple equilibria. In this case, the QZ method does not give us additional information to select one solution. When indeterminacy of equilibrium arises, we employ the recursive method developed by Cho and Moreno (2002). They solve the model forward recursively and propose a selection criterion which is stationary and real-valued by construction. The recursive method is de-
scribed in Appendix B.2. In it, agents coordinate in an equilibrium which yields a unique vector of self-fulfilling expectations. This equilibrium imposes a transversality condition that distant future expectations converge to their long run mean. The remaining expectations are discarded, since agents deem them incapable of being satisfied. Hence, we will use the QZ and recursive methods jointly in order to determine the solution to our macroeconomic system.

4 Data and Estimation

We use quarterly data which spans the period between the second quarter of 1957 and the first quarter of 2001. We present estimates with two measures of inflation: CPI and GDPD. The results obtained using PCE inflation were very similar to those under CPI inflation. The Federal funds rate is the monetary policy instrument. Our results are similar using the 3 month T-Bill rate. We use output detrended quadratically. The results are robust to the use of a linear trend or the Hodrick-Prescott filter. The data is annualized and in percentages. CPI and Federal funds rate data were obtained from Datastream, and both the real GDP and GDPD inflation were obtained from the National Income and Product Accounts (NIPA).

We estimate the structural parameters using Full Information Maximum Likelihood (FIML) by assuming normality of the structural errors. Our FIML estimation procedure allows us to obtain the structural parameters and the VAR reduced-form in one stage, affording a higher efficiency than two-stage instrumental variables techniques. It seems adequate to estimate the whole model jointly, given the simultaneity between the private sector and the Central Bank behavior, as explained by Leeper and Zha (2000).

The log likelihood function can be written as:

\[
\ln L(\theta | X_T, X_{T-1}, ..., X_1) = \sum_{t=2}^{T} \left[ -\frac{3}{2} \ln 2\pi - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} (X_t - \Omega X_{t-1})' \Sigma^{-1} (X_t - \Omega X_{t-1}) \right]
\]

(12)

If the solution falls into the indeterminacy region, the recursive method ignores the possibility of the sunspot shocks discussed in Farmer and Guo (1994). In the case of indeterminacy, our equilibrium can be seen as a sunspot equilibrium without sunspots. As it will be shown below, we obtain multiple equilibria in the first subsample. Lubik and Schorfheide (2002) allow for sunspot shocks in their estimation and cannot reject the existence of a sunspot equilibrium without sunspots in the pre-Volcker period.
where \( \bar{X}_t = X_t - EX_t \), \( \theta = (\delta, \lambda, \mu, \rho, \beta, \gamma, \sigma_{AS}^2, \sigma_{IS}^2, \sigma_{MP}^2) \) and \( \Sigma = \Gamma D \Gamma' \). \( EX_t \) is the unconditional expectation of \( X_t \).

The matrices \( \Omega \) and \( \Gamma \) can be calculated by the QZ method or the recursive method. We maximize the likelihood function with respect to the structural parameters in \( \theta \), not the reduced-form ones in \( \Omega \) or \( \Gamma \). Given the structural parameters, the matrices \( \Omega \) and \( \Gamma \) must be calculated at each iteration. This requires checking whether there is a unique, real-valued stationary solution at each iteration. Whenever there are multiple solutions at the \( i \)-th iteration, we apply the recursive method to select one solution. We choose the initial parameters from the values used in the literature. In order to check for robustness of our estimates we set up different initial conditions, randomizing around the obtained parameter estimates five times. In all of the cases convergence to the same parameter estimates was attained. We also found that the estimates obtained through our recursive method converge to the \( c, \Omega \) and \( \Gamma \) matrices obtained through the QZ method.

5 Dating the Structural Break in the U.S. Economy

Since our strategy consists of accounting for the drop of inflation volatility by the changes in shocks and in propagation, we need to identify two separate subsamples. To this end, we perform a structural break date test, which detects the most likely break date of all the coefficients of an unrestricted VAR over the whole sample period. The idea is that variations in these coefficients reflect changes in the parameters of our underlying structural model. Bernanke, Gertler, and Watson (1997) and Clarida, Galí, and Gertler (1999), among others, have shown evidence of parameter instability across different sample periods.

We use the Sup-Wald test derived by Bai, Lumsdaine, and Stock (1998), which detects the most likely structural break date in the reduced-form coefficients of a vector autoregression. Our motivation for the use of this test is twofold. First, breaks in reduced-form coefficients must come from shifts in structural parameters. In order to respond to the Lucas critique, we need then to split the full sample at the time of the structural break in vector autoregressive coefficients. Second, there is evidence of a change in the unconstrained VAR coefficients which is responsible for the decrease in the overall inflation volatility over the last 20 years (see Ahmed, Levin, and Wilson (2002) and section 6.3
In this respect it seems then adequate to focus on breaks in all the unconstrained parameters.

Table 2 reports the Sup-Wald test associated with unconstrained VARs of orders one to five using the CPI inflation rate. Except for the VAR(1), the beginning of the 4th quarter of 1980 is identified as the most likely break date for the parameters of the reduced-form relation (in the case of the VAR(1), the break date selected is the third quarter of 1980). Since the Schwarz criterion selects the VAR(3) as the order which provides the best fit to the data, we set the beginning of the fourth quarter of 1980, one year after Paul Volcker became Federal Reserve chairman, as our break date. Figure 2 graphs the time series of the Wald statistics for the VAR(3). This break date is robust across inflation and output gap measures and significant at the 1% level. The 90% confidence interval is very tight, including only three quarters. This date coincides with the largest increase, between two quarters, in the average Federal funds rate during the whole sample: From 9.83% in the 3rd quarter of 1980 to 15.85% on the 4th.

While there appears to be a clear break date in the relation among our three macroeconomic variables, it seems plausible that more than one structural break has occurred in the joint fluctuations of inflation, the output gap and the Federal funds rate over the complete sample period. Stock and Watson (2002), for instance, perform a battery of univariate and multivariate tests and conclude that the most likely break date test for the majority of the macroeconomic series is around 1984. In order to gauge the robustness of our break date, we perform the following experiment: We estimate unconstrained VARs for the two subsamples separated by the original break date. Then, with the residuals of these vector autoregressions, we run the Sup-Wald test for both samples. If no other clear structural break dates existed, no obvious break dates should arise in this exercise, since the sample splitting would make the unconstrained parameters approximately stable across samples. Table 3 shows the break date statistics for unconstrained VARs for the two subsamples and Figure 3 graphs the time series of Wald statistics. While the years 1974 and 1986 appear as candidates for break dates across subsamples, these breaks

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4One major difference, however, is that when testing for an unknown break date in a multivariate framework, they restrict their attention to the break in the mean of the GDP growth. Accordingly, they perform the Sup-Wald test on VARs with different components of the GDP, but do not include inflation or the Federal funds rate. Additionally, in our case, we let all the VAR coefficients break whereas they focus on breaks in the unconditional variances.

5Note that we trim the initial and final 15% of the sample when running the Sup-Wald test. As Maddala and Kim (1998) point out, it is customary to do so in order to rule out breaks around the ends.
are not as clear as in the original case, since the exact quarter differs for each VAR order. Given this finding and the fact that two relatively large subsamples will be available for estimation, we will proceed with our analysis assuming that there was a single structural break on the 4th quarter of 1980.

6 Results

In this section we present our main findings. First, we report the U.S. FIML baseline parameter estimates for both inflation specifications and we perform a parameter stability study. Second, we analyze the properties of the implied Rational Expectations equilibrium and the model’s goodness of fit. Then we proceed to explain the influence of the different propagation mechanisms (monetary policy and remaining model’s parameters) on the decline of inflation volatility.

6.1 Parameter Estimates

6.1.1 Baseline Estimates

Table 4 reports the U.S. FIML estimates with both inflation measures. In order to accommodate the documented change in the deterministic trend growth of output (see, for instance, Orphanides and Porter (1998)) we allow for separate quadratic trends across subsamples, just as in Ireland (2001).

The estimates in Table 4 have all the right sign across specifications and most of them are statistically significant. In the AS equation, agents put more weight on expected inflation than on past inflation in both periods, whereas in the IS equation they put around the same weight on the expected and past output gap across periods. The coefficient on the real rate in the IS equation, $\phi$, and the Phillips curve parameter, $\lambda$ are however imprecisely estimated. Estrella and Fuhrer (1999), Smets (2000), Kim (2000) and Ireland (2001) also obtained small and insignificant estimates for these two parameters. Nelson and Nikolov (2002) show that Bayesian and Minimum Distance methods yield larger values estimates of $\phi$ than those obtained through Maximum Likelihood or Instrumental Variables estimators.
The estimates of the monetary policy reaction function reflect the smoothing behavior of the Fed, as the persistence coefficient, $\rho$, is of large magnitude. They also show that the Fed reacted more strongly to future inflation in the second period, although not significantly so, and that it acted in a countercyclical fashion, as $\gamma$ has positive signs in all cases. It is interesting to note that $\gamma$ is significant in the first period across inflation specifications, but in the second period it only becomes significant under the GDPD specification.

Three major stylized facts emerge from Table 4 across specifications. First, the three standard deviations of the structural shocks are lower in the second period, especially the one corresponding to the IS shock. Blanchard and Simon (2001) and Ahmed, Levin, and Wilson (2002) report decreases in their output equation innovations of a very similar magnitude. Cogley and Sargent (2002) also report a 40% decrease of the variance of the shock in their unemployment equation. Stock and Watson (2002) also present evidence that structural shocks have been milder since 1984. Second, the probability distribution of the Fed’s reaction to expected inflation shifted to the right in the second period, but the difference across estimates is not statistically significant. In this respect, the evidence is mixed across studies. On the one hand, Clarida, Gali, and Gertler (1999), with single equation GMM estimation and both Lubik and Schorfheide (2002) and Cogley and Sargent (2002), with a Bayesian MLE approach in a system framework, find significant increases in the Fed reaction to inflation. On the other hand, Sims (1999) and Sims and Zha (2002), with regime switching models and both Ireland (2001) and Cho and Moreno (2002), through frequentist MLE in a system framework, do not find a significant increase. Third, private agents put more weight on expected inflation in the AS equation during the second period. This is more pronounced in the estimation with CPI inflation. Less attention has been paid to this third fact, however. The exception is Boivin and Giannoni (2003), who also report an increase in this parameter.

As a robustness check, we exclude the first 4 observations of the second sample and estimate the model parameters. As Table 4 shows, the stylized facts mentioned above do not change. In fact, none of the results reported below is altered if we exclude the initial observations of the second sample.

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6Cho and Moreno (2002) further show in their small sample analysis that $\beta$ is upwardly biased.
6.1.2 Structural Estimates in the AS and IS equations

So far we have presented the results for the baseline estimates. The appendix shows however that the four baseline parameters of the AS and IS equations ($\delta$, $\lambda$, $\mu$ and $\phi$) are a function of five deeper structural parameters ($\vartheta$, $\varphi$, $\psi$, $\sigma$ and $h$). As can be seen, without further restrictions, we cannot uniquely identify the structural parameters. Our strategy is then to restrict $\psi = 1$ so that the remaining parameters can be identified. The associated standard errors can be computed through the delta-method.

Table 5 presents the structural parameter estimates. As the appendix shows, the price-setters who do not adjust optimally, index their prices with respect to past inflation. They implement the following indexation rule: $\log P_t = \log P_{t-1} + \vartheta \pi_{t-1}$. As a result, $\vartheta$ reflects the degree of indexation with respect to past inflation. Our CPI estimates imply that $\vartheta$ was 0.82 in the first period, and 0.69 in the second, whereas the GDPD imply 0.86 in the first period, and 0.79 in the second. Our estimates are statistically significant and consistent with the implied upper bound of 0.5 for the backward looking term.$^7$ These estimates are similar to those found by Galí and Gertler (1999). They reflect less indexing with respect to past inflation on the side of price-setters. $\varphi$ reflects the probability of not adjusting prices optimally on a given period. It is estimated to be around 0.95, but not significantly in most of the cases.

The implied estimates of the curvature parameter in the utility function, $\sigma$, are between 35 and 112, but are not significantly different from zero across periods or inflation measures. Finally, the habit persistence parameter, $h$, is around 1 and statistically significant across inflation measures and sample periods. Fuhrer (2000) found it to be 0.80.

6.1.3 Parameter Stability Tests

Table 7 presents the Wald tests for parameter stability of the baseline parameters. It shows that at the 5% level, in the case of the estimates with CPI inflation, two parameters reject the null of stability: $\delta$, the forward-looking parameter in the AS equation and $\sigma_{IS}$, the structural IS or demand shock. Precisely these two parameters will be crucial in lowering inflation and output gap volatility, respectively. Interestingly, in their time

$^7$Notice however that this analysis understates the true standard errors, as we calibrate the subjective discount factor.
varying parameter model, Cogley and Sargent (2002) fail to reject the time invariance hypothesis in the inflation equation, which is the only equation where they detect a reasonably high power in their sup-Wald test. In the context of our model, the move towards a more forward-looking price setting seems to be instrumental in producing instability of reduced-form parameters.

In the case of the GDPD estimation, $\delta$ is not significantly different across periods, but $\sigma_{AS}$ and, marginally, $\gamma$, are. We will show that the decline of the AS shocks in the second period triggered the lower GDPD inflation volatility in the second period.

Table 8 shows that, out of the structural parameter estimates in the AS and IS equation, only the $\vartheta$ in the CPI specification rejects the null of stability. Of course, this has to do with the significative difference of $\delta$ across periods.

6.2 Model’s Implied Equilibrium and Goodness of Fit

Table 6 reports the generalized eigenvalues associated with the Rational Expectations equilibrium in both subsamples for the three data specifications. Whereas the second period equilibrium is unique in all cases, the first period estimates give rise to multiple equilibria, as there are more than three eigenvalues less than unity. Under Ricardian fiscal policy, multiple equilibria can arise due to the violation of the “Taylor principle”, whereby the Fed does not stabilize inflation fluctuations ($\beta < 1$). Then, for the first subsample, we select the solution implied by the recursive method, which selects the equilibrium associated with the three smallest eigenvalues.

Table 9 compares, across sample periods, the volatilities of the variables found in the data with their model's counterparts. Since the structural model is nested in a VAR(1) system, all the elements of the implied variance-covariance matrix of the model $(V(X_t))$ can be easily computed from the Rational Expectations model solution in (5) as:

$$\text{vec}(V(X_t)) = (I - \Omega \otimes \Omega)^{-1}(\Gamma \otimes \Gamma)\text{vec}(D)$$

where $I$ is the identity matrix of dimension $9 \times 9$, $\otimes$ is the Kronecker product operator and vec represents an operator stacking the columns of a matrix. All the volatilities are

\[8\] Alternatively, Rudebusch (2003) presents a careful statistical exercise showing that shifts in policy rules by themselves have a small impact on the unconstrained VAR coefficients.
matched with precision except in the case of the second period interest rate volatility. This seems to be due to the highly non-linear behavior of the Federal funds rate during the beginning of the 80s under the Volcker disinflation. Cogley and Sargent (2002) show however that the inclusion stochastic volatility in their time-varying parameter model does not affect the model’s estimates appreciably.

Figure 4 compares the sample autocorrelation functions with those implied by our structural model under the CPI inflation specification. Very similar results were obtained using GDP inflation. In both periods, the sample autocorrelation functions of inflation fall within the model’s confidence bands except for distant autocorrelations in the first period. The model’s output gap autocorrelation seems to overpredict its sample counterpart in both periods. Finally, the model matches the interest rate autocorrelation function across sample periods quite closely. The cross-correlations are not as precisely matched as the autocorrelations. This seems to be due to the fact that two important parameters which capture cross-coefficients feedback (φ and λ) are not precisely estimated.

We now compare the propagation mechanism implied by our model to that of an unconstrained VAR(1). For ease of exposition, we write our model solution alongside the VAR(1) in demeaned form:

\[
X_t = \Omega X_{t-1} + \varsigma_t \tag{14}
\]
\[
X_t = \Omega^{ols} X_{t-1} + v_t \tag{15}
\]

where \(\varsigma_t = \Gamma \epsilon_t\). Under the null of the model, \(\Omega = \Omega^{ols}\). Figure 3 compares the VAR(1) and model’s impulse response functions of the macro variables to the three reduced-form shocks (inflation, output gap and interest rate shocks). It shows that the model does a good job matching the dynamics found in the data along most dimensions for the two sample periods. The model does not reproduce, however, the increase in inflation following an interest rate shock. This is due to the way monetary policy operates in our New-Keynesian economy: an increase in the interest rate lowers the output gap and inflation contemporaneously through the IS and Phillips curve relations. The model also seems to understate the impact of monetary policy on the output gap, especially in the first period. This appears to be related to the small estimate of the coefficient on the real rate in the IS equation, \(\phi\).
The cross-equation restrictions implied by the model are rejected by a likelihood-ratio test. This is mainly due to the strong restrictions embedded in the variance-covariance matrix of the structural errors. Additionally, the model does not reproduce the “price puzzle”, present in most empirical VARs. However, Cho and Moreno (2002) perform a small sample study of the likelihood ratio test of this model and find that when exogenous correlation is added to the model, this is only marginally rejected.

6.3 Explaining the Drop in Inflation Volatility

In this subsection we attempt to determine the sources of the increased stability in the inflation rate in the context of our New-Keynesian macro model. To this end, we develop a counterfactual analysis given the parameter estimates obtained across sample periods. We also develop a sensitivity analysis to assess the robustness of our results. We compare the contribution of both shocks and the model’s propagation mechanism to the decline in inflation volatility. Then, we compare the role of shocks and propagation in the change of each component of inflation volatility: anticipated and unanticipated. Finally we focus our attention on the specific roles of monetary policy authority and the private sector.

6.3.1 Shocks or Propagation? What Propagation?

In this section we study the role of exogenous shocks and internal propagation in the documented inflation volatility drop. Table 10 compares the standard deviations of inflation and the output gap for all the sample combinations of structural shocks and propagation. It performs the analysis for both the empirical VAR(1) and the structural model. Let \( D_i \) be the matrix of structural shocks in period \( i \) and \( \Phi_j \) the matrix of propagation coefficients of period \( j \). Then, for instance, \( \sigma_k(D_i, \Phi_j) \), where \( k = \pi, y, r \) and \( i, j = 1, 2 \), denotes the standard deviation of the variable \( k \) implied by the system including the shocks of sample \( i \) and the propagation of sample \( j \). There are 5 possible counterfactual comparisons which can be carried out:

1. If \( \sigma_k(D_1, \Phi_1) > \sigma_k(D_1, \Phi_2) \), the changes in propagation contribute to a lower volatility of variable \( k \).

2. If \( \sigma_k(D_1, \Phi_1) > \sigma_k(D_2, \Phi_1) \), the changes in shocks contribute to a lower volatility
of variable $k$.

3. If $\sigma_k(D_2, \Phi_2) < \sigma_k(D_1, \Phi_2)$, the changes in shocks contribute to a lower volatility of variable $k$.

4. If $\sigma_k(D_2, \Phi_2) < \sigma_k(D_2, \Phi_1)$, the changes in propagation contribute to a lower volatility of variable $k$.

5. If $\sigma_k(D_1, \Phi_2) < \sigma_k(D_2, \Phi_1)$, changes in propagation are more important than changes in shocks in explaining a lower volatility of variable $k$. To see this, suppose that the four previous inequalities hold. In that case, both shocks and volatility contributed to lower volatility. To determine which factor was more influential, we compare the volatilities implied by the more stabilizing propagation and the larger shocks with the destabilizing propagation and the smaller shocks.

Whereas the first and second inequalities describe how, given an initial subsample, changes in propagation or shocks would affect the volatilities, the third and fourth inequalities reflect the changes in volatilities that would be brought about by returning to past scenarios of shocks or propagation. The fifth comparison allows us to gauge the overall importance of shocks relative to propagation in the (relevant) case that both shocks and propagation contributed to a lower inflation variance in a given period.

Table 10 presents the results of the counterfactual exercise. It shows that the model can explain the lower inflation volatility of the second period. In the case of the CPI, the implied second period inflation volatilities are statistically smaller in the second period. A comparison between $\sigma_\pi(D_1, \Phi_1)$ and $\sigma_\pi(D_1, \Phi_2)$ for both inflation specifications reveals that the changes in propagation in the second period contributed to the decline of inflation volatility. This result is confirmed by the fact that $\sigma_\pi(D_2, \Phi_2) < \sigma_\pi(D_2, \Phi_1)$. The lower shocks also contributed to lower inflation volatility as $\sigma_\pi(D_2, \Phi_1) < \sigma_\pi(D_1, \Phi_1)$ and $\sigma_\pi(D_1, \Phi_2) > \sigma_\pi(D_2, \Phi_2)$.

In the case of the CPI, the change in propagation was more influential than the decline of shocks to reduce overall inflation volatility, since $\sigma_\pi(D_2, \Phi_1) < \sigma_\pi(D_1, \Phi_2)$. As for the decrease in GDPD inflation volatility, the model’s counterfactuals give more importance to the smaller shocks in the second period than to the changes of propagation. Table 9 presents an analogous counterfactual exercise with an unconstrained VAR(1). The results are very similar to those yielded by the structural model.
reveals two main differences in the estimates under GDPD inflation with respect to CPI inflation: First, the increase in the forward-looking component in the AS equation is more moderate in the case of the GDPD estimation. Second, the decline in the standard deviation of the AS shock under GDPD inflation is, in percentage terms, larger than under CPI inflation. In the next subsection we examine the sources of differences across inflation measures.

Table 11 performs a sensitivity analysis to determine the robustness of our findings based on the model. The sensitivity analysis is motivated by the fact that the estimates of the transmission mechanism were imprecisely estimated at very low values. Indeed, Galí and Gertler (1999) have shown that estimates of the Phillips curve using marginal costs instead of the output gaps are larger and significant. Accordingly, we fix the Phillips curve parameter value, $\lambda$, and the coefficient on the real rate, $\phi$ across sample periods at the average of the estimates and also at larger values. Then we estimate the rest of the model’s parameters and compare $\sigma_\pi(D_2, \Phi_1)$ with $\sigma_\pi(D_1, \Phi_2)$. The results remain intact for the three inflation specifications.

Our structural model has the advantage that it reveals along what dimensions propagation changed. In order to gauge the influence of each parameter change in the overall decrease of inflation volatility, Table 12 performs a counterfactual exercise: It calculates the inflation variance which would obtain under the second period estimates of one of the parameters and the first period values of the remaining parameters. Table 12 also shows it shows that the more aggressive response of the Fed to expected inflation in the second period also contributed to the lower inflation volatility. However, for the two data specifications, the most influential individual parameter change was the increase in the forward-looking component of the price setting. This more flexible price setting may have been the result of an increased flexibility in indexation schemes of wage and financial contracts. However, we are also open to the possibility that it is related to monetary policy in some form not specified by current New-Keynesian models.

Panel B of Table 12 develops an analogous exercise to determine the decline of which structural shock was more influential in the decrease of inflation volatility. In both cases, the decline of the AS shock results in a larger decline of volatility. It also shows that in the GDPD specification inflation volatility is more sensitive to the decrease in the size of the AS shocks.
Table 13 analyzes the influence of the private sector behavior and the monetary policy authority on the decline of CPI inflation volatility. To this end, it compares the counterfactual inflation volatility under the first period private sector parameters (IS and AS parameters) and the second period monetary policy rule with that under second period private sector of first period policy rule. We perform this analysis with different parameter combinations. First, we use the baseline parameter estimates. Second, we fixed \( \lambda \) and \( \phi \) at the two period average and estimated the rest of the parameters. Third, we fixed \( \lambda \) and \( \phi \) at values one order of magnitude larger than those found in estimation. In the three cases the changes in the private sector structural parameters were more important in the decline of inflation volatility than those in the monetary policy rule.

As for the output gap volatility, the reduction that we are explaining in our sample is fairly small. Nevertheless, the key factor underlying this small output gap volatility drop is to be found in the smaller shocks, since \( \sigma_y(D_2, \Phi_1) < \sigma_y(D_1, \Phi_2) \) in our two data specifications. The changes in propagation did not contribute to this lower volatility, as \( \sigma_y(D_1, \Phi_1) < \sigma_y(D_1, \Phi_2) \) and \( \sigma_y(D_2, \Phi_1) < \sigma_y(D_2, \Phi_2) \). The decrease in output gap volatility was mostly induced by the significant decrease in the IS shock, which falls significantly in the three specifications. McConnell and Quirós (1992) attribute the smaller output volatility since 1984 to the improvement of inventory management. Such event would enter in our model in the form of smaller structural shocks, since it does not arise endogenously in our New-Keynesian setup. This result is consistent with Simon (2000), Blanchard and Simon (2001), Ahmed, Levin, and Wilson (2002) and Stock and Watson (2002) who, with alternative methodologies, also find that the key factor behind the drop of output volatility was the smaller shocks of the 80s and 90s.

### 6.3.2 Differences between CPI and GDPD inflation rates

In order to gain intuition about the different behavior of the GDPD, it can be useful to summarize the three main differences among the GDPD and the CPI: First, the CPI includes the price of imported goods, unlike the GDPD. Second, the GDPD includes the price of goods purchased by investors, the government, and by foreign buyers of domestic goods, unlike the CPI. Finally, the CPI is a fixed price index whereas GDPD accounts for the changes in the domestic production and consumption, respectively. We do not believe that our finding is related to differences in the weights of the indexes, since the
CPI is a fixed price index but the PCE is not, and we obtained similar results for both. It seems that divergences in coverage are driving the different results. In particular, the GDPD is the broader index in scope since it includes the prices of goods purchased by firms, government, investors and foreign buyers.

Figure 6 graphs all the GDPD inflation components, except consumption, against GDPD inflation. It also shows the graph of imports inflation. Three features are worthwhile mentioning. First, all of the series move close to GDPD inflation except imports. Second, except for government expenses inflation, the remaining series reach their peak in the first quarter of 1974, around the first oil shock. This is also the case for GDPD inflation, but not for CPI or PCE inflation. Third, there is a sharp decline in the private domestic investment inflation around 1980.

We investigate the second moments of these inflation measures in Table 14. We split the sample according to our estimated subsamples. All of the series experience an important drop in volatility in the second period. Interestingly, only domestic private investment has a larger first order correlation in the second period. This should explain why the second period correlation of the GDPD is similar to the first one, unlike the CPI or the PCE.

We further study two components of the fixed investment index (the largest by far within the private domestic investment category): Structures (ST) and Equipment and Software (ES). Figure 7 reveals two important details. First, the sharp drop of prices in the early 80s is much more pronounced in the case of the ST series. On the other hand, the oil crisis effect produced an unusually large spike in ES. Table 14 presents the second moments of ST and ES. It shows that the ST series has a much larger correlation in the first period. It also shows that the decline in correlation of ES is smaller than that of other series such as government expenditures or consumption.

7 Conclusion

In this paper we showed that the more stabilizing propagation mechanisms of the 80s and 90s played a key role in the decline of CPI inflation volatility. The decrease of GDPD

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We are currently estimating the model with the different series of inflation components. Preliminary evidence suggests that investment and exports are causing the differences between CPI and GDPD.
volatility, however, was more influenced by the smaller shocks. We also showed that the leading factor behind the “improved” propagation mechanism was the more forward-looking price setting of the 80s and 90s. In the context of our New-Keynesian model, we showed that the shift towards a more aggressive monetary policy also mattered, but to a lesser extent.

This paper raises a number of questions for future research, but perhaps the most pressing one is related to the contemporaneous increase in the Fed’s responsiveness to inflation and the private sector’s forward-looking behavior in the AS equation. A more forward-looking price setting can be rationalized by several factors, such as an increased flexibility in wage indexation schemes or the development of information technologies which increases both price competitiveness and flexibility. However, as Woodford (2002) observes, variations in agents’ price setting behavior are exogenous in standard AS specifications with endogenous persistence, such as the one employed in this paper. It could be that the price setting behavior of firms is directly related to the Fed’s stance against inflation. Dotsey, King, and Wolman (1999), for example, derive a state-dependent pricing specification. The present paper underscores the need to model and estimate the links between the price setting behavior and the monetary authority’s degree of activism more explicitly.

Another area of future research will be the introduction of monetary aggregates in the structural model. It is well known (see, for instance, Bernanke and Mihov (1998)) that during some periods the Fed targeted money stocks. If this is the case, standard Taylor-rule type estimates could be biased by not considering this fact. By introducing money market clearing we could easily take into account shocks to money demand in the monetary policy rule by adding the money demand as a new equation. Finally, Leeper and Roush (2002) show that the monetary transmission mechanism is not confined to changes in the real rate of interest. Expanding the current demand equation to account for the influence of money supply on output seems a worthwhile exercise.
Appendix

A A Macro Model for the US Economy

A.1 IS Equation

The representative agent seeks to maximize its lifetime expected utility given by:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \psi^t \left[ u \left( C_t, C_{t-1}; \xi_t \right) \right] \right\}
\]

(16)

\( \psi \) is the time discount factor and \( C_t \) is a consumption index of each of the differentiated goods defined by:

\[
C_t = \left[ \int_0^1 C_{t, \bar{C}} \, dt \right]
\]

(17)

The utility function exhibits external habit persistence. \( \xi_t \) is an i.i.d. process which represents disturbances to the preferences. The optimal intertemporal consumption choice is then given by the standard Euler equation:

\[
\frac{1}{1 + \bar{r}_t} = E_t \left\{ \frac{\psi u_c \left( C_{t+1}, C_t; \xi_{t+1} \right)}{u_c \left( C_t, C_{t-1}; \xi_t \right)} \frac{P_t}{P_{t+1}} \right\}
\]

(18)

In the steady state, \( \xi_t = 0 \). We perform a log-linear approximation to (18) and use the market clearing condition, \( \bar{y}_t = C_t \) to substitute for consumption and obtain the IS equation.\(^\text{[11]}\)

Parameterizing the utility function in a standard way,

\[
u \left( C_t, C_{t-1}; \xi_t \right) = \xi_t \frac{1}{1 - \sigma} \left( \frac{C_t}{C_{t-1}^{\bar{C}}} \right)^{1-\sigma}
\]

(19)

we obtain that:

\[
y_t = \mu_1 E_t y_{t+1} + \mu_2 y_{t-1} - \phi \left( r_t - E_t \bar{\pi}_{t+1} \right) + g_t - E_t g_{t+1}
\]

(20)

\(^{[11]}\)Consumption and output expressed as deviations from their balanced growth paths.
where

\[ \mu_1 = \frac{\sigma}{\sigma(1 + h) - h} \]  \hspace{1cm} (21) \\
\[ \mu_2 = \frac{\sigma(1 - h)}{\sigma(1 + h) - h} \]  \hspace{1cm} (22) \\
\[ \phi = \frac{1}{\sigma(1 + h) - h} \]  \hspace{1cm} (23) \\
\[ \phi = \frac{1}{\sigma(1 + h) - h} \]  \hspace{1cm} (24) 

where \( h \) is the habit persistence parameter. Notice that \( \mu_1 + \mu_2 = 1 \). \( \sigma = \frac{u_c \bar{y}}{u_c} \) denotes the inverse of the intertemporal elasticity of substitution, \( \bar{y} \) is the steady state detrended output and \( g_t = \frac{u_c \xi_t}{u_c(\sigma(1 + h) - h)} \xi_t \). All the variables are expressed in percentage deviations from their steady-state values so that \( y_t = \log \left( \frac{\tilde{y}_t}{\bar{y}} \right) \). Since \( \xi_t \) is i.i.d. distributed, we finally obtain:

\[ y_t = \mu_1 E_t y_{t+1} + \mu_2 y_{t-1} - \phi (r_t - E_t \pi_{t+1}) + \epsilon_t^{IS} \]  \hspace{1cm} (25)

### A.2 AS Equation

For ease of exposition, we first present the AS equation derived in the absence of endogenous persistence.

In order to set up an explicit price optimization problem, Calvo (1983) and the subsequent literature assume monopolistic competition in the intermediate product markets. A retail distributor combines the differentiated output of a continuum of monopolistically competitive firms, \( Y_{i,t} \), into a detrended composite product, \( \tilde{y}_t \), with elasticity of substitution between goods \( \theta_t > 1 \):

\[ \tilde{y}_t = \left[ \int_0^1 Y_{i,t}^{\theta - 1} di \right]^{\frac{\theta}{\theta - \theta}} \]  \hspace{1cm} (26)

The demand for each firm \( i \) is obtained by the usual expression (see Blanchard and
Kiyotaki (1987):

\[ Y_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\theta} y_t \]  

(27)

where \( P_{i,t} \) is the price of firm \( i \) and \( P_t \) is the aggregate index defined as:

\[ P_t = \left[ \int_0^1 P_{i,t}^{1-\theta} di \right]^{\frac{1}{1-\theta}} \]  

(28)

In the Calvo (1983) pricing framework, firms maximize their profits according to an arrival rate \((1 - \phi)\). Thus, each firm resets prices every period with a probability \((1 - \phi)\). The firms which do not adjust leave the price unchanged.

Using the law of large numbers, the price index becomes:

\[ P_t = [(1 - \phi)P_{i,t}^{\phi} + \phi P_{t-1}^{\phi}]^{\frac{1}{\phi}} \]  

(29)

where \( P_{i,t}^{\phi} \) is the optimal reset price. Log-linearizing this expression yields:

\[ \pi_t = \frac{1 - \phi}{\phi} \hat{p}_t \]  

(30)

where \( \hat{p}_t = \log \left( \frac{P_{i,t}}{P_t} \right) \).

The optimal dynamic price-setting problem becomes:

\[ \max_{P_{i,t}} \mathbb{E}_t \left[ \sum_{T=t}^{\infty} \psi^{T-t} M_{i,T}^c \Pi_{i,T} \right] \]  

(31)

where \( M_{i,T}^c \) is the stochastic discount factor for contingent claims, where

\[ M_{i,T}^c = \frac{\psi^{T-t} u_c(C_T, C_{T-1}; \xi_T) P_t}{u_c(C_t, C_{t-1}; \xi_t) P_T} \]  

(32)

In particular, the demand for an intermediate good \( i \), \( C_{i,t} \), maximizes the function \( \left[ \int_0^1 C_{i,t}^{\phi+1} di \right]^{\theta} \) subject to the budget constraint \( \int_0^1 P_{i,t} C_{i,t} di = I \), where \( I \) is the agent’s income. Then the market clearing condition \( y_t = C_t \) (and \( Y_{i,t} = C_{i,t} \)) is imposed.
and \( \Pi_{i,T} \) is the profit of firm \( i \) defined as:

\[
\Pi_{i,T} = \left[ \frac{P_{i,T}}{P_T} - s_T \right] \left[ \frac{P_{i,T}}{P_T} \right] ^{-\theta} \bar{y}_T
\]

(33)

where \( s_T \) is the real marginal cost defined as \( s_T = \frac{w_T}{\frac{\partial y_{i,t}}{\partial y_{i,t}}} \). The first order condition associated with the maximization problem in (31) can be expressed as:

\[
E_t \left\{ \sum_{T=t}^{\infty} (\varphi \psi)^{T-t} \left[ (1 - \theta) P_{i,t}^{\theta} P_T^{\theta-1} \right] \left[ \frac{P_{i,t}^{\ast}}{P_t} \right] \right\} = 0
\]

(34)

where \( \mu = \frac{\theta}{\theta - 1} \) is the constant markup. Log-linearizing (34) around the steady state and solving for \( \hat{p}_t = \log \left( \frac{P_{i,t}^{\ast}}{P_t} \right) \), we obtain:

\[
\hat{p}_t = (1 - \varphi \psi) \sum_{T=t}^{\infty} (\varphi \psi)^{T-t} E_t [\hat{s}_T] + \sum_{T=t+1}^{\infty} (\varphi \psi)^{T-t} E_t [\pi_T]
\]

(35)

where \( \hat{s}_t \) is the percentage deviation from steady state of the real marginal cost of producing \( y_{i,t} \). Subtracting \( \varphi \psi \hat{p}_{t+1} \) from both sides of the last equation and using (30) yields a relation describing the inflation dynamics:

\[
\pi_t = \psi E_t \pi_{t+1} + \lambda \hat{s}_t
\]

(36)

where \( \lambda = \frac{(1 - \varphi)(1 - \varphi \psi)}{\varphi} \). Notice the key role of the nominal rigidities linking the real sector of the economy with inflation. Without a time-varying markup, it can be shown that in equilibrium there is a proportionality relation between real marginal costs and the output gap. We then rewrite our AS equation as:

\[
\pi_t = \psi E_t \pi_{t+1} + \lambda y_t
\]

(37)

To add endogenous persistence, we assume that the price-setters who do not adjust optimally, index their prices taking into account previous inflation. Hence,

\[
\log P_t = \log P_{t-1} + \vartheta \pi_{t-1}
\]

(38)
This implies the following price index:

\[ P_t = \left[ (1 - \varphi)P_{t,t}^{1-\theta} + \varphi \left( \frac{P_{t-1}^{1+\theta}}{P_{t-2}} \right)^{1-\theta} \right]^{1-\theta} \]  (39)

Solving the model in analogous way to the case without endogenous persistence, you can obtain the following AS equation:

\[ \pi_t = \delta_1 E_t \pi_{t+1} + \delta_2 \pi_{t-1} + \lambda y_t \]  (40)

where:

\[ \delta_1 = \frac{\psi}{1 + \psi \vartheta} \]  (41)
\[ \delta_2 = \frac{\theta}{1 + \psi \vartheta} \]  (42)

For a time discount factor arbitrarily close to unity, \( \delta_2 \approx 1 - \delta_1 \). In this instance, the supply specification is consistent with the natural rate hypothesis. We also add an exogenous AS shock to the supply equation which accounts for deviations of real wages from their equilibrium value or simply pricing errors. Therefore, the supply equation becomes:

\[ \pi_t = \delta E_t \pi_{t+1} + (1 - \delta)\pi_{t-1} + \lambda y_t + \varepsilon_{t}^{AS} \]  (43)
A.3 Monetary Policy Rule

The instrument of the monetary authority, the Federal funds rate, is set according to the following reaction function:

\[ r_t = \rho r_{t-1} + (1 - \rho) r^*_t + \varepsilon_{MP_t} \]  \hspace{1cm} (44)

\[ r^*_t = \bar{r}^* + \beta (E_t \pi_{t+1} - \bar{\pi}) + \gamma y_t \]  \hspace{1cm} (45)

\( \bar{\pi} \) is the long run equilibrium level of inflation, \( \bar{r}^* \) is the desired nominal interest rate and \( \varepsilon_{MP_t} \) is the monetary policy shock. There are two parts to the equation: The lagged interest rate captures the well known tendency of the Federal Reserve towards smoothing interest rates, whereas \( r^*_t \) represents the “Taylor rule” whereby the monetary authority reacts to deviations of expected inflation from its target and to deviations of output from its potential level. Hence, the monetary policy equation becomes:

\[ r_t = \alpha_{MP} + \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \varepsilon_{MP_t} \]  \hspace{1cm} (46)

where \( \alpha_{MP} = (1 - \rho)(\bar{r}^* - \beta \bar{\pi}) \)

B Rational Expectations Equilibrium

B.1 The QZ Method

In this appendix we derive the Rational Expectations solution of our New-Keynesian model using the generalized (QZ) Decomposition. For ease of exposition we reproduce equations (4) and (5) in mean deviation, so that \( \bar{X}_t = X_t - E X_t \):

\[ B_{11} \bar{X}_t = A_{11} E_t \bar{X}_{t+1} + B_{12} \bar{X}_{t-1} + \epsilon_t \]  \hspace{1cm} (47)

\[ \bar{X}_{t+1} = \Omega \bar{X}_t + \Gamma \epsilon_{t+1} \]  \hspace{1cm} (48)

We further assume that the error terms are serially correlated, i.e. \( \epsilon_t = F \epsilon_{t-1} + w_t \). Our goal is to solve for \( \Omega \) and \( \Gamma \), since they completely determine the equilibrium dynamics.

27
of our system.

Substituting equation (48) into equation (47) we can obtain

$$A_{11}[\Omega(\Omega \bar{X}_{t-1} + \Gamma \epsilon_t) + \Gamma F \epsilon_t] = B_{11}[\Omega \bar{X}_{t-1} + \Gamma \epsilon_t] - B_{12} \bar{X}_{t-1} - \epsilon_t$$  \hspace{1cm} (49)

Collecting the $X_{t-1}$ and $\epsilon_t$ terms yields, respectively:

$$A_{11} \Omega^2 = B_{11} \Omega - B_{12}$$ \hspace{1cm} (50)

$$A_{11} \Omega \Gamma + A_{11} \Gamma F = B_{11} \Gamma - I$$ \hspace{1cm} (51)

Expressing equation (50) in matrix companion form:

$$\begin{bmatrix} A_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Omega^2 \\ \Omega \end{bmatrix} = \begin{bmatrix} B_{11} & -B_{12} \\ I & 0 \end{bmatrix} \begin{bmatrix} \Omega \\ I \end{bmatrix}$$ \hspace{1cm} (52)

Define the $2n \times 2n$ matrices $A = \begin{bmatrix} A_{11} & 0 \\ 0 & I \end{bmatrix}$ and $B = \begin{bmatrix} B_{11} & -B_{12} \\ I & 0 \end{bmatrix}$ where $n$ is the number of endogenous variables. Then the generalized Schur decomposition guarantees the existence of invertible matrices $Q$ and $Z$ such that $QAZ = S$ and $QBZ = T$, with $S$ and $T$ triangular. The ratios $\frac{\lambda}{s_{ii}}$ are the generalized eigenvalues of the matrix pencil $B - \lambda A$, where $\lambda \in \mathbb{C}$ is a given generalized eigenvalue. Premultiply (52) by $Q$, define $H = Z^{-1}$ and apply the QZ decomposition to obtain:

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \Omega^2 \\ \Omega \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \Omega \\ I \end{bmatrix}$$ \hspace{1cm} (53)

where the submatrices $S_{ij}, H_{ij}$ and $T_{ij}$ are of dimension $n \times n$. The second row can be written as:

$$S_{22}(H_{21}\Omega + H_{22}) \Omega = T_{22}(H_{21}\Omega + H_{22})$$ \hspace{1cm} (54)

Equation (54) is satisfied for an $\Omega$ such that:

$$\Omega = -H_{21}^{-1}H_{22}$$ \hspace{1cm} (55)
Finally $\Sigma$ is obtained directly from equation (51):

$$\vec(\Sigma) = [I + F' \otimes (A_{11}\Omega - B_{11})^{-1}A_{11}]^{-1}\vec[(A_{11}\Omega - B_{11})^{-1}] \quad (56)$$

### B.2 The Recursive Method

We can characterize the stationarity, uniqueness and real-valuedness of the equilibrium of our system as follows: If all the eigenvalues, $t_{ii}$, are less than unity in absolute value, $\Omega$ is stationary. If the number of stable generalized eigenvalues is the same as that of the predetermined variables (3 in our model, the lagged endogenous variables), then there exists a unique solution. If there are more than 3 stable generalized eigenvalues, we have multiple solutions. Conversely, if there are less than 3 stable eigenvalues, there is no stable solution. Finally, $\Omega$ is real-valued if (a) each one of its eigenvalues is real-valued, or (b) for every complex eigenvalue of $\Omega$, the complex conjugate is also an eigenvalue of $\Omega$. Unfortunately, in the case of multiple stationary solutions, there seems to be no agreement about the selection of a solution among the candidates. In this case, we use the recursive method developed by Cho and Moreno (2002), who solve the model forward recursively and propose an alternative simple selection criterion which is bubble-free and real-valued by construction. The idea is to construct sequences of convergent matrices, \( \{C_k, \Omega_k, \Gamma_k, k = 1, 2, 3, \ldots\} \) such that:

$$\bar{X}_t = C_k E_{t} \bar{X}_{t+k+1} + \Omega_k \bar{X}_{t-1} + \Gamma_k \varepsilon_t \quad (57)$$

We characterize the solution that is fully recursive as follows. We check first whether $\Omega^* \equiv \lim_{k \to \infty} \Omega_k$ and $\Gamma^* \equiv \lim_{k \to \infty} \Gamma_k$ exist, and $\Omega^*$ is the same as one of the solutions obtained through the QZ method. For the limit to equation (57) to be a bubble-free solution, $\lim_{k \to \infty} C_k E_{t} \bar{X}_{t+k+1}$ must be a zero vector. Then the solution must be of the form:

$$\bar{X}_t = \Omega^* \bar{X}_{t-1} + \Gamma^* \varepsilon_t \quad (58)$$

Finally, we check whether $\lim_{k \to \infty} C_k E_{t} \bar{X}_{t+k+1} = \lim_{k \to \infty} C_k \Omega^{*k} = 0$ using equation (58).

\[\text{13}\] Blanchard and Kahn (1980) suggest the choice of the 3 smallest eigenvalues and McCallum (1999) suggests the choice that would yield $\Omega = 0$ if it were the case that $B_{12} = 0$. Uhlig (1997) observes that McCallum’s criterion is difficult to implement but it often coincides with Blanchard and Kahn’s criterion.
References


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Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CPI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{\pi} )</td>
<td>4.7383</td>
<td>3.5835</td>
<td>3.8939</td>
<td>3.2688</td>
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<tr>
<td>(0.7243)</td>
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<td>(0.6083)</td>
<td>(0.2521)</td>
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<td>( \sigma_\pi )</td>
<td>3.7771</td>
<td>2.0340</td>
<td>3.0329</td>
<td>1.4445</td>
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<td>(0.4687)</td>
<td>(0.3754)</td>
<td>(0.4248)</td>
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<tr>
<td>( \rho_\pi )</td>
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<td>0.5868</td>
<td>0.7444</td>
<td>0.3804</td>
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<td>(0.0654)</td>
<td>(0.1134)</td>
<td>(0.0856)</td>
<td>(0.0918)</td>
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<tr>
<td><strong>PCE</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{\pi} )</td>
<td>4.3062</td>
<td>3.2049</td>
<td>3.6724</td>
<td>2.8020</td>
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<tr>
<td>(0.6025)</td>
<td>(0.3073)</td>
<td>(0.5458)</td>
<td>(0.2477)</td>
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<td>3.0452</td>
<td>1.6318</td>
<td>2.0966</td>
<td>1.2286</td>
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<td>(0.2650)</td>
<td>(0.4219)</td>
<td>(0.1250)</td>
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<td>0.7162</td>
<td>0.8672</td>
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<td>(0.0445)</td>
<td>(0.0671)</td>
<td>(0.0642)</td>
<td>(0.0890)</td>
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<td><strong>GDPD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{\pi} )</td>
<td>4.3584</td>
<td>3.1967</td>
<td>3.8594</td>
<td>2.5841</td>
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<td>(0.5601)</td>
<td>(0.3812)</td>
<td>(0.5481)</td>
<td>(0.1972)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_\pi )</td>
<td>2.7266</td>
<td>1.8127</td>
<td>2.4958</td>
<td>0.8215</td>
</tr>
<tr>
<td>(0.5601)</td>
<td>(0.3812)</td>
<td>(0.3768)</td>
<td>(0.0923)</td>
<td></td>
</tr>
<tr>
<td>( \rho_\pi )</td>
<td>0.9983</td>
<td>0.9203</td>
<td>0.9874</td>
<td>0.9484</td>
</tr>
<tr>
<td>(0.0360)</td>
<td>(0.0332)</td>
<td>(0.0507)</td>
<td>(0.0389)</td>
<td></td>
</tr>
</tbody>
</table>

This Table shows the descriptive statistics of CPI inflation (CPI), PCE inflation (PCE) and GDP Deflator inflation (GDPD). \( \bar{\pi} \) stands for the average, \( \sigma_\pi \) is the standard deviation and \( \rho_\pi \) is the first order autocorrelation. These statistics and their respective standard errors (in parentheses) were computed using generalized method of moments (GMM) estimation. The weighting matrix is constructed using 3 Newey-West lags. The following system of equations was estimated for each inflation measure:

\[
\begin{align*}
e_{1t} & = \pi_t - \bar{\pi} \\
e_{2t} & = (\pi_t - \bar{\pi})^2 - \sigma_\pi^2 \\
e_{3t} & = (\pi_t - \bar{\pi})(\pi_{t-1} - \bar{\pi}) - \rho_\pi(\pi_{t-1} - \bar{\pi})^2
\end{align*}
\]

where \( \bar{\pi} \) is the sample mean of inflation. \( e_{1t}, e_{2t} \) and \( e_{3t} \) are the disturbances so that \( e_t = \{e_{1t}, e_{2t}, e_{3t}\} \) and \( E[e_t] = 0 \). There are three parameters to be estimated and three orthogonality conditions, so that the system is exactly identified.
### Table 2: Sup-Wald Break Date Statistics

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>VAR</th>
<th>Sup-Wald</th>
<th>Break Date</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
</table>

This Table lists the Sup-Wald values of the break date test derived by Bai, Lumsdaine, and Stock (1998). The test detects the most likely break date of a break in all of the parameters of unconstrained VARs of orders 1 to 5. The Table shows the results of the test using the CPI, quadratically detrended output gap and the Federal funds rate.
## Table 3: Sup-Wald Break Date Statistics (Robustness Test)

### Panel A: 1st subsample

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>VAR</th>
<th>Sup-Wald</th>
<th>Break Date</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
</table>

### Panel B: 2nd subsample

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>VAR</th>
<th>Sup-Wald</th>
<th>Break Date</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
</table>

Panel A lists the Sup-Wald values of the break date test derived by Bai, Lumsdaine, and Stock (1998) applied to a VAR(3) of the first subsample residuals implied by the vector autoregression of CPI inflation, the output gap and the Federal funds rate. Panel B lists the values associated with the analogous exercise for the second subsample.
This Table shows the FIML parameter estimates of the structural New-Keynesian macro model with CPI and GDPD inflation, respectively. Output is detrended quadratically and the Federal funds rate is used as interest rate. The subsample associated with 1st P. spans the period 1957:2Q-1980:3Q, 2nd P. spans the period 1980:4Q-2001:1Q and 2nd P.-gap spans 1981:4Q-2001:1Q. The model’s equations in demeaned form are:

\[
\begin{align*}
\pi_t & = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda y_t + \epsilon_{AS,t} \\
y_t & = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (r_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \\
r_t & = \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{MP_t}
\end{align*}
\]
Table 5: **Structural Parameters in the AS and IS equations**

<table>
<thead>
<tr>
<th></th>
<th>CPI 1st P.</th>
<th>CPI 2nd P.</th>
<th>GDP Defl. 1st P.</th>
<th>GDP Defl. 2nd P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vartheta$</td>
<td>0.8242</td>
<td>0.5934</td>
<td>0.8598</td>
<td>0.7873</td>
</tr>
<tr>
<td></td>
<td>(0.0332)</td>
<td>(0.0505)</td>
<td>(0.0281)</td>
<td>(0.0310)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.9150</td>
<td>0.9699</td>
<td>0.9645</td>
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<tr>
<td></td>
<td>(0.3900)</td>
<td>(0.6668)</td>
<td>(0.7566)</td>
<td>(0.5155)</td>
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<tr>
<td>$\sigma$</td>
<td>35.4795</td>
<td>111.34</td>
<td>51.0918</td>
<td>75.3125</td>
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<tr>
<td></td>
<td>(30.6540)</td>
<td>(106.4391)</td>
<td>(27.2027)</td>
<td>(71.4929)</td>
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<td>$h$</td>
<td>0.9575</td>
<td>1.0507</td>
<td>1.0171</td>
<td>1.0892</td>
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<tr>
<td></td>
<td>(0.0862)</td>
<td>(0.1085)</td>
<td>(0.0910)</td>
<td>(0.1482)</td>
</tr>
</tbody>
</table>

This Table shows the structural parameters of the AS and IS equations. Standard errors appear in parentheses and are computed through the delta-method.
Table 6: **Generalized Eigenvalues: Baseline Specifications**

<table>
<thead>
<tr>
<th>Panel A: CPI</th>
<th>1st period</th>
<th>2nd period</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ1</td>
<td>0.7484</td>
<td>0.5935</td>
</tr>
<tr>
<td>ξ2</td>
<td>0.8319-0.1245i</td>
<td>0.9129 - 0.0372i</td>
</tr>
<tr>
<td>ξ3</td>
<td>0.8319+0.1245i</td>
<td>0.9129 + 0.0372i</td>
</tr>
<tr>
<td>ξ4</td>
<td>0.9978</td>
<td>1.0081</td>
</tr>
<tr>
<td>ξ5</td>
<td>1.1234</td>
<td>1.0843</td>
</tr>
<tr>
<td>ξ6</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: GDPD</th>
<th>1st period</th>
<th>2nd period</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ1</td>
<td>0.8430</td>
<td>0.7822</td>
</tr>
<tr>
<td>ξ2</td>
<td>0.8644-0.0991i</td>
<td>0.8990-0.0429i</td>
</tr>
<tr>
<td>ξ3</td>
<td>0.8644+0.0991i</td>
<td>0.8990+0.0429i</td>
</tr>
<tr>
<td>ξ4</td>
<td>0.9955</td>
<td>1.0117</td>
</tr>
<tr>
<td>ξ5</td>
<td>1.1326</td>
<td>1.1194</td>
</tr>
<tr>
<td>ξ6</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

This Table reports, across sample periods, the generalized eigenvalues which determine the stability of the structural macro model under the two data specifications.
Table 7: **Wald Tests of Parameter Stability. Baseline Parameters**

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP Defl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ</td>
<td>4.6075</td>
<td>0.8954</td>
</tr>
<tr>
<td></td>
<td>(0.0318)</td>
<td>(0.3440)</td>
</tr>
<tr>
<td>λ</td>
<td>1.3261</td>
<td>0.0226</td>
</tr>
<tr>
<td></td>
<td>(0.2495)</td>
<td>(0.8805)</td>
</tr>
<tr>
<td>µ</td>
<td>0.7120</td>
<td>0.2097</td>
</tr>
<tr>
<td></td>
<td>(0.3988)</td>
<td>(0.6470)</td>
</tr>
<tr>
<td>φ</td>
<td>0.5898</td>
<td>0.1905</td>
</tr>
<tr>
<td></td>
<td>(0.4425)</td>
<td>(0.6625)</td>
</tr>
<tr>
<td>ρ</td>
<td>3.0666</td>
<td>0.0351</td>
</tr>
<tr>
<td></td>
<td>(0.0799)</td>
<td>(0.8514)</td>
</tr>
<tr>
<td>β</td>
<td>2.0980</td>
<td>2.7163</td>
</tr>
<tr>
<td></td>
<td>(0.1475)</td>
<td>(0.0993)</td>
</tr>
<tr>
<td>γ</td>
<td>0.0590</td>
<td>3.8374</td>
</tr>
<tr>
<td></td>
<td>(0.8081)</td>
<td>(0.0501)</td>
</tr>
<tr>
<td>σ_{AS}</td>
<td>2.3259</td>
<td>20.8547</td>
</tr>
<tr>
<td></td>
<td>(0.1272)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>σ_{IS}</td>
<td>26.0413</td>
<td>11.9314</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>σ_{MP}</td>
<td>0.2468</td>
<td>1.5944</td>
</tr>
<tr>
<td></td>
<td>(0.6193)</td>
<td>(0.2067)</td>
</tr>
</tbody>
</table>

This Table shows the Wald-test statistics of parameter instability for the baseline parameters. The probability values of no structural change appear in parentheses. The Wald statistic used is: \( W = (\theta_1 - \theta_2)'(V_1 + V_2)^{-1}(\theta_1 - \theta_2) \). Andrews and Fair (1988), show that it is distributed as a chi-square with \( p \) degrees of freedom under the null of parameter stability.
Table 8: Wald Tests of Parameter Stability. Structural Parameters

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDP Defl.</th>
</tr>
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<td>$\vartheta$</td>
<td>14.5842</td>
<td>3.0025</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
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<td>$\varphi$</td>
<td>0.0051</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.9431)</td>
<td>(0.9920)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.4691</td>
<td>0.1003</td>
</tr>
<tr>
<td></td>
<td>(0.4934)</td>
<td>(0.7515)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.4523</td>
<td>0.1719</td>
</tr>
<tr>
<td></td>
<td>(0.5012)</td>
<td>(0.6784)</td>
</tr>
</tbody>
</table>

This Table shows the Wald-test statistics of parameter instability for the structural estimates in the AS and IS equations. The probability values of no structural change appear in parentheses. The Wald statistic used is:

$$W = (\theta_1^p - \theta_2^p)'(V_1^p + V_2^p)^{-1}(\theta_1^p - \theta_2^p).$$

Andrews and Fair (1988), show that it is distributed as a chi-square with $p$ degrees of freedom under the null of parameter stability.
Table 9: **Standard Deviations**

Panel A: CPI

<table>
<thead>
<tr>
<th></th>
<th>1st period</th>
<th></th>
<th>2nd period</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample</td>
<td>Model</td>
<td>Sample</td>
<td>Model</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>3.78</td>
<td>3.64</td>
<td>2.09</td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td>[2.86 4.70]</td>
<td>[2.87 5.41]</td>
<td>[1.37 2.81]</td>
<td>[1.54 3.44]</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>2.62</td>
<td>2.28</td>
<td>2.50</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>[2.25 2.98]</td>
<td>[1.68 4.37]</td>
<td>[2.06 2.93]</td>
<td>[1.05 4.39]</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>2.94</td>
<td>2.78</td>
<td>3.18</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>[2.12 3.76]</td>
<td>[1.70 5.13]</td>
<td>[1.94 4.42]</td>
<td>[1.59 4.76]</td>
</tr>
</tbody>
</table>

Panel B: GDPD

<table>
<thead>
<tr>
<th></th>
<th>1st period</th>
<th></th>
<th>2nd period</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample</td>
<td>Model</td>
<td>Sample</td>
<td>Model</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>2.73</td>
<td>2.83</td>
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<td>1.32</td>
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<td>[2.16 3.29]</td>
<td>[2.06 5.25]</td>
<td>[0.92 2.71]</td>
<td>[1.01 3.01]</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>2.62</td>
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<td>2.50</td>
<td>2.16</td>
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<td>[2.25 2.98]</td>
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<td>[2.06 2.93]</td>
<td>[1.05 4.39]</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>2.94</td>
<td>2.78</td>
<td>3.18</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>[2.12 3.76]</td>
<td>[1.70 5.13]</td>
<td>[1.94 4.42]</td>
<td>[1.59 4.76]</td>
</tr>
</tbody>
</table>

This Table reports both the sample and model standard deviation across sample periods and data specifications. The volatilities’ standard errors appear in brackets. The sample standard errors were obtained through the GMM estimation outlined in the note of Table 1. The empirical standard errors were computed through the following Montecarlo procedure: We perform random draws from the asymptotic distribution of the parameter set to construct $\Omega$ and $\Gamma$ matrices of the model’s solution which yields volatility values for $\pi$, $y$ and $r$. We replicate this exercise 1,000 times discarding the non-stationary solutions in the process.
Table 10: **Counterfactual Standard Deviations**

<table>
<thead>
<tr>
<th>Panel A: CPI</th>
<th>Model</th>
<th>(D_1,\Phi_1)</th>
<th>(D_1,\Phi_2)</th>
<th>(D_2,\Phi_1)</th>
<th>(D_2,\Phi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_\pi)</td>
<td>3.64</td>
<td>2.48</td>
<td>2.98</td>
<td>2.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.87 5.41]</td>
<td>[1.88 3.80]</td>
<td>[2.44 5.33]</td>
<td>[1.54 3.44]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>2.28</td>
<td>4.05</td>
<td>1.28</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.68 4.37]</td>
<td>[1.89 4.96]</td>
<td>[1.00 3.44]</td>
<td>[1.05 4.39]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>2.78</td>
<td>3.14</td>
<td>2.12</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.70 5.13]</td>
<td>[2.10 5.31]</td>
<td>[1.35 4.51]</td>
<td>[1.59 4.76]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: GDPD</th>
<th>Model</th>
<th>(D_1,\Phi_1)</th>
<th>(D_1,\Phi_2)</th>
<th>(D_2,\Phi_1)</th>
<th>(D_2,\Phi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_\pi)</td>
<td>2.83</td>
<td>2.25</td>
<td>1.66</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.06 5.25]</td>
<td>[1.76 4.09]</td>
<td>[1.22 4.24]</td>
<td>[1.01 3.01]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>2.07</td>
<td>3.25</td>
<td>1.39</td>
<td>2.16</td>
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</tr>
<tr>
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<td>[1.50 4.00]</td>
<td>[1.47 4.67]</td>
<td>[0.98 3.22]</td>
<td>[0.94 4.36]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>2.66</td>
<td>3.01</td>
<td>1.86</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.97 5.07]</td>
<td>[1.78 5.10]</td>
<td>[1.44 4.36]</td>
<td>[1.39 4.95]</td>
<td></td>
</tr>
</tbody>
</table>

This Table reports the VAR(1) and model’s implied standard deviations implied by all the combinations of volatilities and propagation. The volatilities in column \(D_i,\Phi_j, i = 1, 2, j = 1, 2\), are those associated with the \(i-th\) period structural error standard deviations and the \(j-th\) period propagation coefficients. The corresponding empirical 95% confidence intervals appear in brackets. They were computed through the Montecarlo procedure outlined in Table 9.
Table 11: Counterfactuals Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>ϕ</td>
<td>β1</td>
<td>γ1</td>
</tr>
<tr>
<td>0.004</td>
<td>0.010</td>
<td>0.96</td>
<td>0.75</td>
</tr>
<tr>
<td>0.010</td>
<td>0.025</td>
<td>1.02</td>
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</tr>
<tr>
<td>0.010</td>
<td>0.050</td>
<td>1.01</td>
<td>0.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>ϕ</td>
<td>β1</td>
<td>γ1</td>
</tr>
<tr>
<td>0.001</td>
<td>0.008</td>
<td>0.90</td>
<td>1.94</td>
</tr>
<tr>
<td>0.005</td>
<td>0.010</td>
<td>0.90</td>
<td>1.06</td>
</tr>
<tr>
<td>0.010</td>
<td>0.025</td>
<td>0.93</td>
<td>1.06</td>
</tr>
</tbody>
</table>

This Table reports a robustness analysis of the Table 10 counterfactuals fixing λ and ϕ across periods around their estimated values. β_i and γ_i represent the estimates of β and γ in period i. Panel A lists the analysis with CPI inflation and Panel B with the GDPD inflation. σ_π(i, j) is the volatility of inflation under the shocks of the i – th period and propagation of the j – th period.
Table 12: Counterfactual Inflation Volatilities

Panel A: Contribution of Model’s Parameters

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\pi(D_1, \Phi_1)$</td>
<td>3.64</td>
<td>2.83</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \delta_2)$</td>
<td>2.47</td>
<td>2.27</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \lambda_2)$</td>
<td>3.82</td>
<td>2.85</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \mu_2)$</td>
<td>3.76</td>
<td>2.84</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \phi_2)$</td>
<td>4.38</td>
<td>2.84</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \rho_2)$</td>
<td>3.54</td>
<td>2.83</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \beta_2)$</td>
<td>3.26</td>
<td>2.72</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \gamma_2)$</td>
<td>3.63</td>
<td>2.81</td>
</tr>
</tbody>
</table>

Panel B: Contribution of Model’s Volatilities

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>GDPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\pi(\theta_1, \sigma_{AS}^2)$</td>
<td>3.06</td>
<td>1.66</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \sigma_{IS}^2)$</td>
<td>3.56</td>
<td>2.82</td>
</tr>
<tr>
<td>$\sigma_\pi(\theta_1, \sigma_{MP}^2)$</td>
<td>3.64</td>
<td>2.83</td>
</tr>
</tbody>
</table>

This Table reports the counterfactual inflation volatilities which would have arisen under the parameter estimates of the first period together with the second period estimate of an individual parameter or volatility.
This Table lists counterfactual standard deviations for different parameter combinations in the model under CPI inflation. \( \sigma_\pi(1 - 2) \) and \( \sigma_\pi(2 - 1) \) are the counterfactual inflation volatilities. They were constructed as follows: \( \sigma_\pi(i - j) \) is the inflation standard deviation computed under the structural parameters of the AS and IS equations of period \( i \) and the monetary policy parameters of period \( j \). The structural shocks were fixed at the two period average. The model’s equations in demeaned form are:

\[
\begin{align*}
\pi_t &= \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \lambda_y t + \epsilon_{AS_t} \\
y_t &= \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi(r_t - E_t \pi_{t+1}) + \epsilon_{IS,t} \\
r_t &= \rho r_{t-1} + (1 - \rho) [\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{MP,t}
\end{align*}
\]
Table 14: Descriptive Statistics: Inflation Components

<table>
<thead>
<tr>
<th></th>
<th>$\pi^g$</th>
<th>$\pi^i$</th>
<th>$\pi^x$</th>
<th>$\pi^m$</th>
<th>$\pi^s$</th>
<th>$\pi^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\pi_{-1}}$</td>
<td>3.38</td>
<td>4.08</td>
<td>4.57</td>
<td>10.85</td>
<td>4.85</td>
<td>3.99</td>
</tr>
<tr>
<td>$\sigma_{\pi_{-2}}$</td>
<td>2.37</td>
<td>2.33</td>
<td>2.70</td>
<td>6.17</td>
<td>3.62</td>
<td>2.58</td>
</tr>
<tr>
<td>$\rho_{\pi_{-1}}$</td>
<td>0.72</td>
<td>0.73</td>
<td>0.58</td>
<td>0.79</td>
<td>0.67</td>
<td>0.87</td>
</tr>
<tr>
<td>$\rho_{\pi_{-2}}$</td>
<td>0.44</td>
<td>0.87</td>
<td>0.53</td>
<td>0.24</td>
<td>0.85</td>
<td>0.82</td>
</tr>
</tbody>
</table>

This Table reports the standard deviations and first-order correlations of the inflation components across subsamples. $\sigma_{\pi_{-1}}$ is, for instance, the first period inflation volatility. $\pi^g$ is the government expenses component, $\pi^i$ is the investment component, $\pi^x$ the exports, $\pi^m$ the imports, $\pi^s$ structures and $\pi^e$ equipment and software.
The top figure graphs the historical series of the CPI, PCE and GDPD inflation rates from 1957:2Q to 2001:2Q. The bottom panel graphs the corresponding rolling standard deviation of the inflation rates. The rolling standard deviations are constructed using a forward looking 20 quarter window.
Figure 2: Series of Wald Statistics: All parameters break for a VAR(3)

of the time series of the Wald statistics which detects a break in all the parameters of an unconstrained VAR(3). The variables in the VAR are CPI inflation, quadratically detrended output and the Federal funds rate. The sample period is 1957:2Q-2001:2Q. The initial and final 15% of the sample are trimmed.

Figure 3: Sup-Wald robustness test

These two figures graph the time series of the Wald statistics which detects a break in all the parameters of an unconstrained VAR(3) of the residuals associated with the vector autoregression of CPI inflation, the output gap and the Federal funds rate. The first subsample spans the period 1957:2Q-1980:3Q and the second subsample covers the period 1980:4Q-2001:2Q. The initial and final 15% of the samples are trimmed.
Figure 4: Model and Sample Autocorrelation Functions

This figure graphs the implied model’s autocorrelations (solid thick lines) together with the sample autocorrelations (dashed lines) using the CPI data specification. The 95% confidence intervals lie within the solid thin lines.
Panel A: 1st Period

Panel B: 2nd Period

Figure 5: Impulse Response Functions to the Reduced Form Shocks

This Figure compares the VAR(1) and model’s impulse response functions of the macro variables to a one standard deviation of the three reduced-form shocks: inflation, output gap and interest rate shock. We report the responses under the CPI specification.
These table graphs the inflation components of the GDPD against the GDPD series from 1957:2Q to 2001:2Q. The solid line corresponds to a given component whereas the dotted line is the GDPD.
These tables graphs the inflation components of the private domestic investment inflation against the GDPD series from 1957:2Q to 2001:2Q. The solid line corresponds to a given component whereas the dotted line is the GDPD.