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Testing of Nonstationary Cycles in Financial Time Series Data

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ABSTRACT

In this article we propose a new method for testing nonstationary cycles in financial time series data. In particular, we use a procedure due to Robinson (1994) that permits us to test unit root cycles in raw time series. These tests have several distinguishing features compared with other procedures. In particular, they have a standard null limit distribution and they are the most efficient ones when directed against the appropriate alternatives. In addition, the procedure of Robinson (1994) allows us to test unit root cycles at each of the frequencies, and thus permits us to approximate the number of periods per cycle. The results, based on the daily structure of the Spanish stock market prices (IBEX 35) show that some intra-year cycles occur, and they take place at approximately 6, 9 or between 24 and 50 periods.

Keywords: Stock market; Unit root cycles; Nonstationarity.
JEL Classification: C22 ; G14.

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1. Introduction

A major theme of financial time series analysis concerns the adequate modelling of stock market prices and, though there is generally agreement about its nonstationary nature, there is still little consensus about its appropriate modelling. Most of empirical literature assume that stock prices posess unit roots and the key question becomes then the analysis of the autocorrelations of the underlying I(0) disturbances. Thus, empirical work has concentrated on stock market returns. The early work found that autocorrelations of daily and weekly returns are close to zero, and concluded that the stock market is efficient (see Fama, 1970). Summers (1986), though, took issue with this interpretation, and stressed that slowly-decaying temporary components in stock prices can generate weak negative correlation in short-horizon returns, but strong negative correlation in long-horizon ones. The latter could either reflect irrationality of the market, or result from time-varying equilibrium expected returns in the presence of rational asset pricing (see Poterba and Summers, 1988). The potential presence of long memory in financial asset returns has also been an important subject of both theoretical and empirical research. If asset returns display long memory, or strong dependence, they exhibit significant autocorrelation between observations widely separated in time and contradicts the weak form of the market efficiency hypothesis, which states that conditioning on historical returns, future asset returns are unpredictable. A number of authors have tested this hypothesis in the stock markets. Greene and Fielitz (1977) used the R/S method and found evidence of persistence in the daily US stock return series. On the contrary, using a modified version of this statistic, Lo (1991) found no evidence to support that hypothesis. Using the modified R/S and other regression methods, Cheung and Lai (1995) found no evidence of persistence in several international stock return series, while Crato (1994) reports similar evidence for the stock returns series of the G-7 countries. The question of long memory in smaller markets has received little attention. Barkoulas et. al (1996) examined the long memory property in the Greek stock market and concluded in favour of the existence of long-term persistence in its behaviour.

Other authors have dealt with the empirical fact that while the returns exhibit little or no autocorrelation, their squares have noticeable one. Attempts to model this phenomenon began with the ARCH(p) model of Engle (1982), followed by its GARCH(p,q) extension (Bollerslev, 1986), and the stochastic volatility model of Taylor (1986), with numerous elaborations of these themes. Robinson (1991) considered extensions of the ARCH(p) and GARCH(p, q) that might entail arbitrarily slow decay of autocorrelations of squared returns, including long memory, where autocorrelations are not summable, and Whistler (1990) applied relevant tests he developed to financial data. Ding and Granger (1996) and others have developed such models further. On the other hand, Andersen and Bollerslev (1997), Breidt et al. (1998), Harvey (1998) considered a long memory version of Taylor`s (1986) stochastic volatility model. Robinson and Zaffaroni (1998) examined a non-linear moving average model whose squares have long memory while Teyssiere (1998) disscussed a variety of ARCH type long memory functional forms involving various forms of nonlinearity.

All these previous works that use long memory models concentrate at the long run or zero frequency and do not examine other frequencies that might be the source of the nonstationarity. In this article, we take a completely different approach and examine the nonstationary nature of prices throughout the cyclical structure by means of using new statistical techniques based on fractional integration at the cyclical frequencies.

The study of cyclical structures in daily financial time series data has not been very much used in previous econometric models and one by product of this work is its emergence as a credible alternative to the classical first differenced (or even fractional) models. There exist different approaches when modelling cycles. Traditionally, deterministic models based on trigonometric functions of time, that were fitted by linear regression techniques, were proposed but they were shown to be inappropriate in many series. Stochastic models, based on stationary autoregressive processes were then proposed (see, eg., Harvey, 1985). However, in many series, the cycles evolve or change over time, and nonstationary cycles have been studied by Ahtola and Tiao (1987). In that paper, they propose a test statistic for testing unit root cycles embedded in autoregressive (AR(2)) processes. Robinson (1994) also develops tests for unit root cycles, however, unlike Ahtola and Tiao (1987), they are not based on autoregressions but on fractional models of the form advocated by Gray et al. (1989, 1994). In a recent article, Gil-Alana (2001a) showed that the tests of Robinson (1994) outperform Ahtola and Tiao (1987) for the unit root cycles in a number of cases.[1] Furthermore, Andersen and Bollerslev (1997) found evidence of strong intraday periodicity in return volatility in foreign exchange and equity models and Robinson (2001) also suggests the use of cyclical structures when modelling financial data.

The outline of this article is as follows: Section 2 briefly presents Robinson's (1994) procedure for testing unit root cycles in raw time series. In Section 3, the tests are

[1] Unit root cycles have also been examined by Chan and Wei (1988) and Gregoir (1999a,b), who derive the limiting distribution of least squares estimates of AR processes with complex-conjugate unit roots, with inference based on parametric estimates. Bierens (2001) also use a model of this sort to test for the presence of business cycles in the annual change of monthly unemployment in the UK

applied to the daily structure of the Spanish stock market prices while Section 4 contains some concluding comments.

2. Testing of unit root cycles

Ahtola and Tiao (1987) proposed tests for unit root cycles which are embedded in an AR(2) process of form:

$$x_t \;=\; \phi_1 \, x_{t-1} \;+\; \phi_2 \, x_{t-2} \;+\; u_t, \qquad t \;=\; 1, 2, ... \qquad (1)$$

which, under the null hypothesis,

$$H_o : \; |\phi_1| \; < \; 2 \quad and \quad \phi_2 \;=\; -1 \qquad (2)$$

becomes the cyclical I(1) model specified below in (5). Gray et al (1989, 1994) extended the unit root model to allow for a fractional degree of integration. In particular, they considered processes like:

$$(1 \,-\, 2\mu L + L^2)^d \, x_t \;=\; u_t, \quad t \;=\; 1, 2, \ldots, \qquad (3)$$

where d can be any real number and where u_t is an I(0) process, defined in the context of the present paper, as a covariance stationary process with spectral density function which is bounded and bounded away from zero at any frequency. Gray et al. (1989) showed that x_t in (3) is stationary if $|\mu| < 1$ and $d < 0.50$ or if $|\mu| = 1$ and $d < 0.25$. They also showed that the polynomial in (3) can be expressed in terms of the Gegenbauer polynomial $C_{j,d}(\mu)$ such that for all real $d \neq 0$:

$$(1 \,-\, 2\mu L \,+\, L^2)^{-d} \;=\; \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j, \qquad (4)$$

where

$$C_{j,d}(\mu) \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k \,(d)_{j-k}\,(2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d)_j \;=\; \frac{\Gamma(d+j)}{\Gamma(d)},$$

where Γ(x) means the Gamma function, and a truncation will be required below (4) to make (3) operational. Thus, the process in (3) becomes:

$$x_t \;=\; \sum_{j=0}^{t-1} C_{j,d}(\mu)\, u_{t-j}, \qquad t \;=\; 1, 2, ...,$$

and when d = 1, we have:

$$x_t \;=\; 2\mu\, x_{t-1} \;-\; x_{t-2} \;+\; u_t, \qquad t \;=\; 1, 2, ..., \tag{5}$$

which is a cyclic I(1) process with the periodicity determined by μ. We can now take $\mu = \cos w_r$, with $w_r = 2\pi/r$, and r will indicate the number of periods required to complete the whole cycle. Examples of simulated realizations of unit root cyclical models like (5) can be found in Gil-Alana (2001b).

Robinson (1994) proposed tests for unit root cycles which are embedded in fractional models of form as in (3). Let's now briefly describe the testing procedure. Following Bhargava (1986), Schmidt and Phillips (1992) and others on parameterization on unit roots, we can consider the regression model,

$$y_t \;=\; \beta' z_t \;+\; x_t, \quad t = 1, 2, ..., \tag{6}$$

where y_t is the raw time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of exogenous regressors, that may include, for example, an intercept ($z_t \equiv 1$), or an intercept and a linear time trend, (i.e., $z_t = (1,t)'$); and the regression errors x_t are of form as in (3). Robinson (1994) developed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d \;=\; d_o, \tag{7}$$

in (6) and (3), for any real value d_o, and thus, also including the unit root model (5) in case of $d_o = 1$. The functional form of the test statistic, denoted by $\hat{s}$, is described in the Appendix.

Based on H_o (7), Robinson (1994) established that under certain regularity conditions:[2]

$$\hat{s} \quad \rightarrow_d \quad N(0,1) \qquad as \quad T \rightarrow \infty, \tag{8}$$

and this standard limit distribution holds across the different types of regressors in z_t in (6) and also across the different types of disturbances u_t in (3). Thus, a one-sided 100α%-level test of H_o (7) against the alternative H_1: $d > d_o$ ($d < d_o$) is given by the rule: 'Reject H_o if $\hat{s} > z_\alpha$ ($\hat{s} < -z_\alpha$)', where the probability that a standard normal variate exceeds z_α is α. Furthermore, he shows that the above tests are efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $d = d_o + \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Other versions of the tests of Robinson (1994), based on annual and seasonal (quarterly and monthly) data can be respectively found in Gil-Alana and Robinson (1997, 2001) and Gil-Alana (1999), and a small application of the present version of the tests is Gil-Alana (2001a).

3. Unit root cycles in the Spanish stock market prices

The time series data analysed in this section correspond to the log-transformation of the daily structure of the Spanish stock market prices, (IBEX 35), obtained for the time period 4-January-1994 to 26-November-2001. The Spanish stock market index IBEX 35 is a

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[2] These conditions are very mild, and concern technical assumptions to be satisfied by $\psi(\lambda)$ in the Appendix.

value-weighted index that includes the thirty five most traded stocks of the Spanish stock market. Every six months the effective trading volumes of all stocks are studied in order to adjust the stocks and their weights that will form the index in the following six months.

Denoting the time series y_t, we employ throughout the model in (3) and (6), with $z_t = (1,t)'$, $t \geq 1$, $(0, 0)'$otherwise, and $\mu = \cos w_r$, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ... \tag{9}$$

$$(1 - 2\cos w_r L + L^2)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{10}$$

testing H_o (7), with $d_o = 1$, for values $w_r = 2\pi/r$, r = 1, 2, …, T/2, thus testing for unit root cycles at all possible frequencies.[3] We treat separately the cases of $\beta_0 = \beta_1 = 0$ a priori, (i.e., including no regressors in the undifferenced regression); β_0 unknown and $\beta_1 = 0$ a priori, (i.e., including an intercept); and β_0 and β_1 unknown (i.e., with an intercept and a linear time trend), and model the I(0) disturbances to be both white noise and to have parametric autocorrelation.

The test statistic reported in Table 1 (and also in Tables 2 – 5) is the one-sided one given by $\hat{s}$ in the Appendix. However, instead of presenting the results for the whole range of values of r, we only report across the tables, those cases where we find at least one non-rejection value for each type of regressors.[4]

(Insert Table 1 about here)

Starting with the case of white noise disturbances, (in Table 1), we see that if we do not include regressors, the unit root null hypothesis cannot be rejected when r is equal to 6 and 9. Including an intercept, the non-rejection values take place at r = 6, 37, 38 and 39,

[3] Note that in case of r = 1, the model reduces to the I(2) hypothesis, with a peak ocurring exclusively at the long run or zero frequency.

[4] The test statistic was also computed based on the first differenced data and the unit root null hypothesis was rejected for all values of r and all type of disturbances.

and if we include an intercept and a linear time trend, the unit root null cannot be rejected at r = 6, 24 and 25. Therefore, the results presented across this table suggest that unit root cycles may be plausible alternative ways of modelling this series, obtaining some evidence of intra-year effects in its behaviour.

However, the significance of the above results may be in large part due to the un-accounted for I(0) autocorrelation in u_t. Thus, in Tables 2 and 3, we also permit AR(1) and AR(2) disturbances. Modelling u_t in terms of an AR(1) process, the null was always rejected in case of no regressors, and including an intercept and/or a linear time trend, we find some non-rejection values, with r ranging between 35 and 38 and between 49 and 53. If u_t is AR(2), the non-rejections take place at r = 6, 35, 36, 50, 51 and 52.

(Insert Tables 2 and 3 about here)

AR modelling of I(0) processes is very conventional, but there exist many other types of I(0) processes, including ones outside the stationary and invertible ARMA class. One that seems especially relevant and convenient in the context of the present tests is that proposed by Bloomfield (1973), in which the spectral density function of u_t is given by:

$$f(\lambda;\tau) \;=\; \frac{\sigma^2}{2\pi}\,\exp\left(2\sum_{r=1}^{m}\tau_r\cos(\lambda r)\right). \tag{11}$$

The intuition behind this model is the following. Suppose that u_t follows an ARMA process of form:

$$u_t \;=\; \sum_{r=1}^{p}\phi_r\,u_{t-r} \;+\; \varepsilon_t \;-\; \sum_{r=1}^{q}\theta_r\;\varepsilon_{t-r},$$

where ε_t is a white noise process and all zeros of $\phi(L) = (1 - \phi_1 L - \ldots - \phi_p L^p)$ lying outside the unit circle and all zeros of $\theta(L) = (1 - \theta_1 L - \ldots - \theta_p L^p)$ lying outside or on the unit circle. Clearly, the spectral density function of this process is then:

$$f(\lambda;\,\tau) \;=\; \frac{\sigma^2}{2\pi}\left|\frac{1-\sum_{r=1}^{q}\theta_r\; e^{i r \lambda}}{1-\sum_{r=1}^{p}\phi_r\; e^{i r \lambda}}\right|^2, \tag{12}$$

where τ corresponds now to all the AR and MA coefficients and σ^2 is the variance of ε_t. Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (11) approximates (12) well where p and q are of small values, which usually happens in economics. Like the stationary AR(p) model, the Bloomfield (1973) model has exponentially decaying autocorrelations and thus we can use a model like this for u_t in (10). Formulae for Newton-type iteration for estimating the τ_l are very simple (involving no matrix inversion), updating formulae when m is increased are also simple, and we can replace $\hat{A}$ in the Appendix by the population quantity:

$$\sum_{l=m+1}^{\infty} l^{-2} \;=\; \frac{\pi^2}{6} \;-\; \sum_{l=1}^{m} l^{-2},$$

which indeed is constant with respect to the τ_j (unlike what happens in the AR case). The Bloomfield (1973) model, has not been very much used in previous econometric models, (though the Bloomfield model itself is a well-known model in other disciplines, see, e.g., Beran, 1993), and one by-product of this work is its emergence as a credible alternative to the fractional ARMA which have become conventional in parametric modelling of I(0) processes.[5] The results of Robinson's (1994) tests based on Bloomfield disturbances (with m = 1, 2) are respectively given in Tables 4 and 5.

(Insert Tables 4 and 5 about here)

[5] Among the few empirical applications found in the literature are Velasco and Robinson (2000) and Gil-Alana (2001a)

Starting with m = 1, we observe a large proportion of non-rejection values compared with previous tables, and the values of r where the unit root null cannot be rejected widely oscillate between 3 and 73. If m = 2, (Table 5), the non-rejections occur with r ranging between 4 and 69. In view of all these results, we can conclude by saying that some intra-year cyclical components are present in the Spanish stock market prices. Thus, the standard approach of taking first differences may be obscuring the presence of other nonstationary components in the cyclical structure of the series.

4. Concluding comments

In this article we have examined the nonstationary behaviour of the Spanish stock market prices (IBEX 35) by means of using new statistical techniques based on unit root cycles. We have used a version of the tests of Robinson (1994) that permits us to test nonstationary cycles at each of the frequencies of the process and thus, it permits us to approximate the number of periods per cycle. The results show that some intra-year cyclical components are present in the data and thus, they may be taken into account when modelling this series. In fact, the lack of non-rejection values when using first differenced data suggests that the standard practice of assuming first differences may be obscuring other important sources of nonstationarity.

The frequency domain approach used in this article seems to be unpopular with many econometricians, and it is important to stress that the results reported across this paper have nothing to do with nonparametric spectral estimation. There exist time domain versions of the test statistics (cf. Robinson, 1991) and the preference here for the frequency domain set-up of Robinson (1994) is motivated by the somewhat greater elegance of formulae it affords, especially when the Bloomfield model is used. In finite samples, the time and frequency domain versions of $\hat{s}$ will differ from each other, in some cases possibly considerably. Under the null, the difference is $O_p(T^{-1/2})$, but substantial differences could appear when the null hypothesis is seriously in error, because of the great degree of noncircularity of nonstationary processes. It is not known in general to what extent this could lead to different conclusions and work in this direction is now in progress.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic or financial data. A natural following-up step would be to test fractional cycles, i.e., allowing d_o in (7) to be a real number rather than 1. Of course, it would also be of interest in this context to estimate the order of integration of the cyclical component of the series. There exist several procedures for estimating the fractional differencing parameter in seasonal and cyclical contexts, (e.g., Ooms, 1997; Arteche and Robinson, 1999, 2000; etc.), however, they are not only computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed. Other questions such as the possible extension of the Bloomfield model to a multivariate setting has yet to be investigated.

Appendix

The test statistic is given by:

$$\hat{s} \; = \; \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad (8)$$

where T is the sample size, and

$$\hat{a} \; = \; \frac{-2\pi}{T} \sum_{j=1}^{*} \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \qquad \hat{\sigma}^2 \; = \; \sigma^2(\hat{\tau}) \; = \; \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j);$$

$$\hat{A} \; = \; \frac{2}{T}\left(\sum_{j=1}^{*} \psi(\lambda_j)^2 - \sum_{j=1}^{*} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{*} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{*} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j) \right)$$

$$\psi(\lambda_j) \; = \; \log\left| 2\left(\cos\lambda_j \; - \; \cos w_r\right)\right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j;\hat{\tau}),$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t \; = \; (1 - 2\cos w_r L + L^2)^{d_o}\, y_t \; - \; \hat{\beta}'\bar{z}_t$, with

$$\hat{\beta} = \left(\sum_{t=1}^{T} \bar{z}_t\, \bar{z}_t' \right)^{-1} \sum_{t=1}^{T} \bar{z}_t\, (1 - 2\cos w_r L + L^2)^{d_o}\, y_t; \quad \bar{z}_t = (1 - 2\cos w_r L + L^2)^{d_o}\, z_t,$$

evaluated at $\lambda_j = 2\pi j/T$ and g is a known function coming from the spectral density function of $\hat{u}_t$: $f(\lambda;\tau) \; = \; \frac{\sigma^2}{2\pi}\, g(\lambda;\tau)$, with $\hat{\tau}$ obtained by minimising $\sigma^2(\tau)$. Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, for example, if u_t is white noise, $g \equiv 1$ and, if u_t is an AR process of form: $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, so that the AR coefficients are function of τ. Finally, the summation on * in the above expressions are over $\lambda \in M$, where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_u - \lambda_u, \rho_u + \lambda_u), \; u = 1, 2, \ldots, s\}$, such that ρ_u, $u = 1, 2, \ldots, s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.

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TABLE 1			
Testing of unit root cycles with white noise disturbances			
r	Type of regressors		
Number of periods per cycle	No regressors	An intercept	An intercept and a linear time trend
6	**0.377'**	**-0.024'**	**-0.068'**
9	**-0.271'**	18.371	17.139
24	-18.082	23.806	**1.276'**
25	-18.079	21.751	**-0.475'**
37	-18.297	**0.879'**	-12.267
38	-18.304	**-0.369'**	-12.766
39	-18.188	**-1.544'**	-13.111

' and in bold: Non-rejection values at the 95% significance level.

TABLE 2			
Testing of unit root cycles with AR(1) disturbances			
r	Type of regressors		
Number of periods per cycle	No regressors	An intercept	An intercept and a linear time trend
35	-58.635	10.061	**1.394'**
36	-52.795	9.712	**0.472'**
37	-48.105	9.254	**-0.431'**
38	-44.333	8.712	**-1.312'**
49	-24.634	**1.301'**	-8.100
50	-23.610	**0.652'**	-8.437
51	-23.135	**0.014'**	-8.889
52	-22.494	**-0.604'**	-9.244
53	-21.802	**-1.206'**	-9.543

' and in bold: Non-rejection values at the 95% significance level.

TABLE 3			
Testing of unit root cycles with AR(2) disturbances			
r	Type of regressors		
Number of periods per cycle	No regressors	An intercept	An intercept and a linear time trend
6	-12.559	**-0.124'**	**-0.394'**
35	-43.137	17.339	**1.327'**
36	-43.007	16.366	**-0.776'**
50	-41.098	**1.367'**	-21.519
51	-42.634	**0.202'**	-22.933
52	-42.748	**-0.986'**	-23.733

' and in bold: Non-rejection values at the 95% significance level.

TABLE 4			
Testing of unit root cycles with Bloomfield (1) disturbances			
r	Type of regressors		
Number of periods per cycle	No regressors	An intercept	An intercept and a linear time trend
3	**-0.617'**	-17.708	-14.537'
13	-5.367	**-0.001'**	**-1.074'**
14	-4.984	2.447	**1.141'**
44	-12.423	12.368	**0.916'**
45	-12.492	12.046	**0.814'**
46	-12.557	11.746	**0.143'**
47	-12.611	11.506	**-0.544'**
48	-12.644	10.364	**-1.106'**
67	-12.557	**1.250'**	-7.906
68	-12.621	**0.963'**	-7.878
69	-12.620	**0.094'**	-8.214
70	-12.545	**-0.136'**	-8.477
71	-12.641	**-0.348'**	-8.831
72	-12.541	**-1.100'**	-8.652
73	-12.611	**-1.266'**	-8.960

' and in bold: Non-rejection values at the 95% significance level.

TABLE 5			
Testing of unit root cycles with Bloomfield (2) disturbances			
r	Type of regressors		
Number of periods per cycle	No regressors	An intercept	An intercept and a linear time trend
4	**0.578'**	**-0.415'**	2.500
20	-5.165	**1.150'**	-1.840
21	-6.249	**-0.269'**	**-1.402'**
22	-4.559	**0.092'**	-2.403
23	-5.372	**0.690'**	**-1.411'**
24	-3.030	**0.174'**	4.125
34	-6.274	6.714	1.346
56	-8.996	7.021	**0.367'**
57	-9.152	10.395	**1.344'**
58	-9.297	8.429	**-0.041'**
61	-8.437	5.618	**0.722'**
62	-9.574	9.781	**-0.353'**
63	-9.234	7.702	**-1.302'**
68	-9.813	8.331	**0.028'**
69	-9.685	6.856	**-0.838'**

' and in bold: Non-rejection values at the 95% significance level.